Competing for Time: A Study of Mobile Applications*

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Abstract

Apps compete for limited user time (budget competition) regardless of their functions. Complementary apps are gross substitutes when budget competition dominates functional competition. I estimate a discrete-continuous demand model incorporating a binding time constraint, using overlapping user data from China in 2017. I use updates to disentangle complementarity from correlated preferences. An easy-to-compute index is proposed to gauge budget competition and

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validated against the structural model. Although negligible in 2017, budget competition can increase by tens or hundreds of times due to its quadratic nature and the remarkable growth of star apps (video, gaming, and superapps).

We're competing with sleep, on the margin.

–Reed Hastings, Netflix CEO, 2017

1 Introduction

When asked about competition during a Netflix earnings call in 2017, Reed Hastings responded with the the opening quote (Hern, 2017). This is a new perspective on competition. He elaborated on this perspective on another occasion (Raphael, 2017), stating, "Think about if you didn't watch Netflix last night: What did you do? There's such a broad range of things that you did to relax and unwind, hang out, and connect–and we compete with all of that." Hastings highlighted the role of time constraint in competition analysis. In this paper, I refer to it as "budget competition" to distinguish it from "functional competition" captured by complementarity or substitutability. Users have at most 24 hours per day. A minute spent on Tik Tok is a minute not spent on WeChat. Similarily, housing and food expenditure would crowd out discretionary expense. Budget compeition is salient in the mobile Internet industry because of the scarcity of time and its concentration (see Figure 1). Budget competition has been invoked in the landmark antitrust case *Qihu v. Tencent* in 2013 to expand the relevant market.[1] Tencent contended that its instant-messaging software QQ competes with all other Internet companies

[1] For a brief introduction of this case, see https://www.pymnts.com/cpi_posts/qihoo-360-v-tencent-first-antitrust-decision-by-the-supreme-court/ and https://enipc.court.gov.cn/en-us/news/view-22.html.

for user attention (time).[2] This claim is trivially true for all apps/software and offline activities regardless of complementarity or substitutability. What matters is the significance of budget competition. When budget competition is substantial, a merger of complements can hurt consumers. To quantify budget competition, we must estimate a structural model of demand with a binding time constraint, in addition to allowing for complementarity/substitutability.

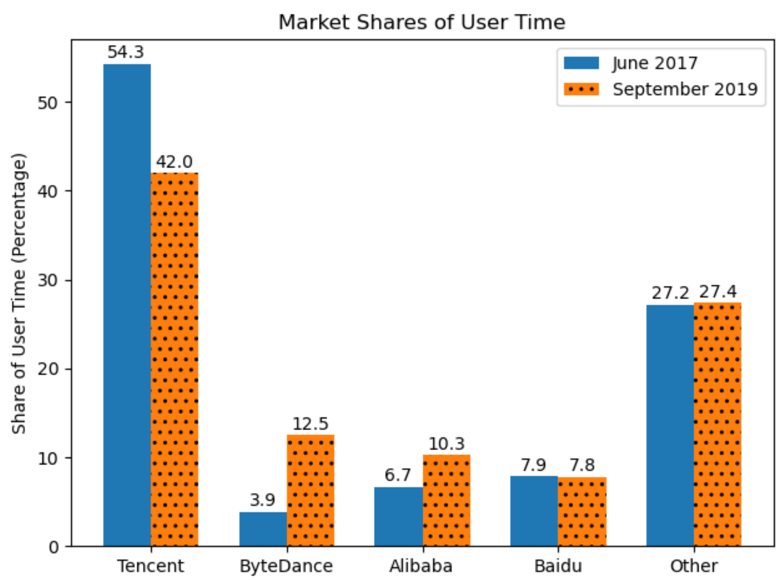


Figure 1: Market Shares of Tech Giants in China
Note: Market shares are calculated based on time spent on apps developed by each tech giants in China.
Data Source: Quest Mobile.

[2]Hence the relevant market in this case should include all major Internet companies and their software. The Supreme People's Court rejected this argument with qualitative analysis in the final adjudication in 2013.

The difficulty in estimating such a model is threefold. First, modeling competition without price is a daunting task. There are no price variations, and therefore we cannot estimate price elasticities. Functional definitions prove inadequate. WeChat, the flagship app of Tencent, is classified as "Social Networking" by the Apple App Store and "Communication" by the Google Play Store as of 2022. Users know that WeChat is more than the two definitions: it is also a mobile payment app, a publishing platform, a platform of mini programs, and so on. It is everything and it is competing with everyone. Second, adding a budget constraint would further complicate utility maximization problems and hence demand estimation. Third, estimating complementarity/substitutability is difficult. We need more than aggregate market share data to separate complements from substitutes. Addressing these challenges necessitates a new demand model.

In this paper, I propose a model of time allocation to apps. The model features a quadratic utility function to capture the discrete-continuous nature of app usage and allow for substitutes as well as complements (Thomassen *et al.*, 2017)[3]. I add a binding time constraint to study budget competition. In this model, an app is described by a taste parameter, a satiation parameter, and interaction parameters with other apps. The taste parameter is the marginal utility at zero usage, and the satiation parameter determines how fast the marginal utility depreciates as an user spends more time on the app. The interaction parameters are cross partial derivatives that describe the functional interdependence/overlapping between apps: if the interaction

[3]Lewbel & Nesheim (2019) also use a quadratic utility model.

parameter between a pair of apps is positive, then they have interdependent functions and are complements; otherwise, they have overlapping functions and are substitutes. Taste parameters have random components that can be correlated across apps. Users allocate their time to apps and offline activities subject to a time constraint. Quadratic utility functions are second-order approximations to any utility functions. Therefore, my model nests the random coefficients discrete choice model of Berry *et al.* (1995) as a special case with only taste parameters.

I formally define budget competition and functional competition. Functional competition captures the fact that if two apps offer similar functions, using one would reduce the (marginal) utility of the other. Budget competition captures the fact that all apps are competing for the limited time of users. Budget competition is relevant for the monetary budget as well. Within the quadratic utility framework, I decompose the gross diversion ratio, the proportion of time diverted to another app due to its exit[4], into two parts: diversion because of functional competition and diversion because of budget competition. An implication of this decomposition is that when budget competition dominates functional competition, complementary apps are gross substitutes. The likelihood of this scenario depends on a nonlinear function of structural parameters. I propose a descriptive index to assess budget competition. The index can be computed with aggregate usage data or a simple survey of users. The index highlights that budget competition between two apps grows with the correlation in their usage and increase **quadratically** with time spent on

[4]The analysis applies to entry and price changes as well.

the apps. Consequently, mergers involving apps with significant time shares or apps targeting the same niche market should trigger scrutiny from antitrust authorities, irrespective of their functional aspects.

I estimate the model using weekly market-level app usage data from China in the first quarter of 2017. Markets in this data set are demographic groups in China defined by age, gender, and province. I observe the number of active users and usage (time spent) of popular apps on Android smartphones. Active user data aids in identifying taste parameters of apps, while usage data aids in identifying satiation parameters. Moreover, for each pair of apps, I observe the number of overlapping users who use both apps in a week. While informative, these overlapping user data are insufficient for identifying complementarity/substitutability. Overlapping users between two apps can be the result of complementarity between the two apps, or the fact that the preferences of the two apps are positively correlated due to unobserved characteristics. I use updates as instruments. Updates of an app should affect the utility of this app but not the *utilities* of other apps. However, updates of an app could change the *usage* of other apps through complementarity/substitutability. This is similar to the strategy used in Gentzkow (2007). I use GMM estimation *a la* Berry *et al.* (1995) such that I could utilize instrument variables (IVs).

My model can recover diverse competition relationships. I apply this model to three representative pairs of apps: a pair of substitutes *a priori* (Baidu Map and Amap), a pair of complements *a priori* (Baidu and Baidu Map), and a pair of apps with an ambiguous relationship (WeChat and Kwai). In each case, the estimated results are reasonable. WeChat and Kwai were weak substitutes though they did not offer similar functions at the time. This can be interpreted as one social media app competing with another social media app. IVs are crucial to my estimation. When I assume away correlated preferences and rely only on the overlapping user data for identification, Baidu Map and Amap are estimated to be almost independent apps.

I then numerically decompose the competition effects of one app on another into functional competition and budget competition using the structural estimates and compare them with the results from the descriptive index. The two sets of results are close in terms of orders of magnitude. Budget competition are negligible (less than 0.02 hours for 1000 smartphone users) for the first two pairs of apps (Baidu Map and Amap, Baidu and Baidu Map). The reason is that the time spent on Baidu Map and Amap is small. Therefore if the apps of interest are "small" (in terms of time spent), researchers can model the demand of these apps without a time constraint and still capture virtually all the competitive effects. The budget competition effect is orders-of-magnitude larger (3.3 hours for 1000 smartphone users) for WeChat and Kwai because user preferences for the two apps are positively correlated, and a large number of users spend a substantial amount of time on both them. 3.3 hours for 1000 smartphone users is still a small number. Because of the spectacular growth of WeChat and Kwai since the first quarter of 2017 and the quadratic nature of budget competition, the budget competition effect is 306.7 hours for 1000 smartphone users for the first quarter of 2013, about 100 times the number in 2017.[5] I use examples (WeChat and Kwai, Netflix

[5]In the first quarter of 2023, time spent on WeChat is about 10 hours per week and time spent on Kwai is about 5 hours per week. According the index, the budget competition

and sleep) to show how the simple index can provide insight into competition. This index can be combined with institutional knowledge (complementarity, binge-watching, multi-stage budgeting, etc) to get more accurate estimates of budget competition. The decomposition highlights that being too "large" *per se* is a source of antitrust concern when analyzing mergers of apps.

This paper contributes to the emerging literature on mobile applications. Due to data limitations, researchers have mostly focused on the supply side of apps (Liu *et al.*, 2013; Yin *et al.*, 2014; Bresnahan *et al.*, 2014b,a; Liu, 2017; Wen & Zhu, 2017; Ershov, 2018; Leyden, 2019). The demand side for apps is either absent or described with aggregate ranking or downloads data from app stores (Carare, 2012; Ghose & Han, 2014; Li *et al.*, 2016; Li & Agarwal, 2017; Yi *et al.*, 2017; Le Guel *et al.*, 2020; Deng *et al.*, 2020).[6] An immediate predecessor of this paper is Han *et al.* (2016). They employ a multi-nominal discrete-continuous extreme value (MDCEV) model developed by Bhat (2005), incorporating correlation in utilities among various applications through a factor analytic structure. With individual level panel data from Nielsen KoreanClick, they estimate positive or negative correlations in preferences among apps. However, substitutes or complements are not modeled in their paper. As the authors have noted in their paper, the correlation of preferences between Naver and Daum and that between Kakao Talk and Kakao Story are estimated to be positive and large. However, the first pair are substitutes

effect is $1000 \times \frac{5\times 10}{168-5} = 306.7$ hours for 1000 users.

[6]Both Wu *et al.* (2022) and Lee (2018) use a panel of individual usage of smartphone. However, both observe usage of categories rather than apps. Lee (2018) estimates the demand for smartphone. Wu *et al.* (2022) uses a hidden Markov model to analyze what motivates mobile app usage.

(two search engines) and the second pair are complements. In contrast, this study explicitly disentangles substitutability/complementarity from correlated preferences using overlapping user data and instrumental variables.

A concurrent paper by Kawaguchi *et al.* (2022) simulate mergers of apps. They estimate demand and supply for apps in two categories with usage and advertising data from Japan. Their paper imposes more restrictive conditions on demand. The disparities between their model and mine underscore the trade-off between flexible competition patterns and scalability. Aridor (2023) addresses the market definition problem in the mobile Internet industry through experimental methods. He disabled access to Instagram and YouTube on participants' phones. Aridor (2023) discovers that a significant share of time is diverted to apps in different categories. This finding corroborates the concept of budget competition outlined in my paper.

Methodologically, this paper extends the framework proposed by Berry *et al.* (1995). This model represents the first attempt to integrate four key components into a consumer demand framework: discrete-continuous decisions, interactions between products, budget constraints, and estimation with instruments. This paper contributes to the literature on the demand for differentiated goods in economics and marketing, particularly focusing on cases where complementarity is of interest (Kim *et al.*, 2002; Nair *et al.*, 2005; Song & Chintagunta, 2006, 2007; Mehta, 2007; Gentzkow, 2007; Thomassen *et al.*, 2017; Ershov *et al.*, 2018; Vélez-Velásquez, 2019; Lewbel & Nesheim, 2019; Wang, 2020). Unlike Gentzkow (2007), this paper accounts for both the extensive margin (selection of products) and the intensive margin (quantities of selected products) in consumer decisions. This distinction is particularly crucial for estimating complementarity. Consumers purchase two boxes of cereal with different flavors due to their preference for variety (which exhibits decreasing marginal utility) rather than complementarity. A discrete choice model employing bundles of various products cannot distinguish between complementarity and taste for variety. Taste for variety is captured through satiation parameters and can be estimated with usage data in this study. Additionally, this paper contributes to the investigation of time allocation within transportation research (Kitamura, 1984; Bhat, 2005; Pawlak *et al.*, 2015, 2017; Bhat, 2018) by directly estimating relationships between activities. Moreover, this model offers a flexible second-order approximation to consumer decisions, allowing for adaptation to explore other research topics.

This paper contributes to the ongoing policy discourse regarding regulating the digital economy (Furman *et al.*, 2019; European Commission. Directorate General for Competition., 2019; Scott Morton *et al.*, 2019). A key challenge in analyzing the digital economy is that the digital economy is characterized by free services, whereas conventional economic tools necessitate pricing data.[7] Complements and substitutes are defined with compensated cross-price elas-

[7] In his opinion piece in the Washington Post, Tim Wu (Wu, 2018) argues:

> "Our standards for assessing mergers, fixated on consumer prices, were a poor match for the tech economy and are effectively obsolete."

In the report commissioned by the Stigler Committee on Digital Platforms, Scott Morton *et al.* (2019) proposes

> "The law needs better analytical tools to take into account the impact of potential and nascent competitors and competition. Market definition will vary according to what consumers are substituting between[......]."

ticities and market power is defined with prices as well. This study employs time variations rather than price variations to model the demand for apps. The decomposition of competition proposed herein introduces a novel theory of harm to consumers: mergers of complementary apps may hurt users if budget competition dominates functional competition. The descriptive index in this paper can also serve as a useful tool for regulators to identify mergers where budget competition may be significant.

2 Model

2.1 The Baseline Model

A consumer $i = 1, 2, , ..., I$ allocates her time T to J apps and an outside option denoted by $j = 0$. The utility from an allocation described by $\mathbf{t} = [t_{i0}, t_{i1}, t_{i2}, ..., t_{iJ}]'$ where t_{ij} is the amount of time allocated to option $j = 0, 1, 2, ..., J$ is given by

$$U(\mathbf{t}) = \boldsymbol{\mu}'\mathbf{t} + 0.5\mathbf{t}'\boldsymbol{\Gamma}\mathbf{t} \tag{1}$$

where

$$\boldsymbol{\mu} = [\mu_{i0}, \mu_{i1}, ..., \mu_{iJ}]'$$

and

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{i0} & \gamma_{i01} & \cdots & \gamma_{i0J} \\ & \gamma_{i1} & \cdots & \gamma_{i1J} \\ & & \ddots & \vdots \\ & & & \gamma_{iJ} \end{bmatrix}.$$

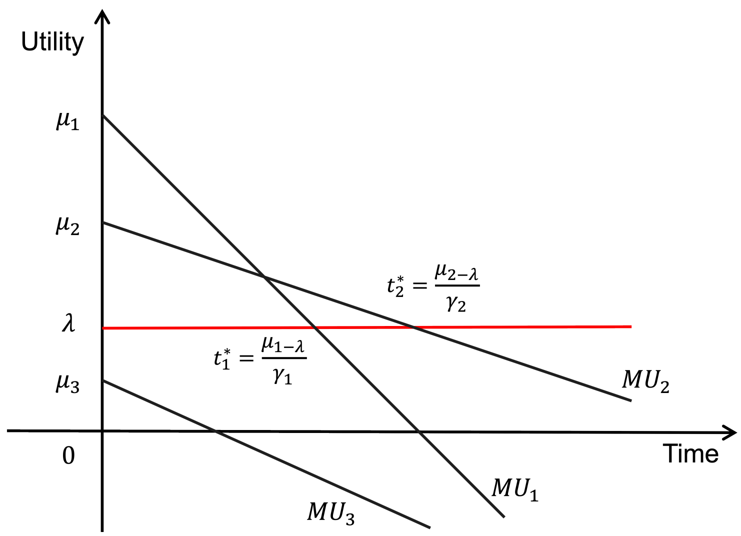
$\boldsymbol{\mu}$ is a $(J+1)\times 1$ vector of first order parameters and $\boldsymbol{\Gamma}$ is a $(J+1)\times(J+1)$ symmetric matrix of second order parameters. The marginal utility of app j is

$$MU_{ij} = \mu_{ij} + \gamma_{ij}t_{ij} + \sum_{j'\neq j}\gamma_{ijj'}t_{ij'}.$$

The marginal utility of app j consists of three components. The first term μ_{ij} is the marginal utility of app j at zero usage and it will be referred to as the taste parameter of app j. γ_{ij} in the second term determines how MU_{ij} changes as a user spends more time on app j. Therefore, γ_{ij} should be negative and will be referred to as the satiation parameter of app j. The last term captures the impact of app j' on app j: if parameter $\gamma_{ijj'} > 0$, then MU_{ij} is increasing in $t_{j'}$ and they are complements; otherwise, they are substitutes. Therefore, the interaction parameter $\gamma_{ijj'}$ determines if j and j' are likely to be used together.

The conventional definition of complements and substitutes[8] hinges on the cross-derivatives of **compensated demand functions**, aligning closely with the substitution effect in classical demand theory (Mas-Colell *et al.*, 1995). For a pair of apps with $\gamma_{jj'} = 0$, their compensated cross derivatives would be positive, making them substitutes by the conventional definition. In other words, any two unrelated products are substitutes because they can substitute each other in providing utilities. This may be at odds with how firms think about competition and substitution. With the notable exception of Netflix, firms usually think about competition in terms of functions and features. In contrast, my approach defines complements and substitutes based on the

[8]Samuelson (1974) discusses various definitions of complements and substitutes.



Note: This graph plots the marginal utility of three apps: app 1, app 2, and app 3. λ is the marginal utility at the optimal allocation. For simplicity, I ignore $\gamma_{jj'}$. MU is the marginal utility of each app.

Figure 2: Marginal Utilities and Optimal Allocation of Time

cross-derivatives of **utility functions**, and hence does not rely on utility maximization or expenditure minimization. This distinction is important for the definition of budget competition in section 3.1.

At the optimal level $\mathbf{t}^*$, the marginal utilities of chosen apps should be equalized. Denote this as λ. Zero usage arises naturally when the marginal utility at zero is too small, i.e., $\mu_{ij} < \lambda$. In Figure 2, I plot three apps with different combinations of μ_j and γ_j with $\gamma_{jj'} = 0$. Intuitively, μ_j determines if an app is used and conditional on being used, γ_j determines the time spent on app j.

I choose the quadratic utility function because it naturally models the discrete-continuous nature of app usage and the complementarity/substitutability between apps. However, it comes at the cost of a quadratically increasing $\boldsymbol{\Gamma}$ matrix size with respect to the number of apps (J). The characteristic-space approach, commonly used to address this issue, projects μ_{ij}, γ_{ij}, and $\gamma_{ijj'}$ onto a lower-dimensional characteristic space with K dimensions. The key assumption is that products of interest are linear combinations of a small set of features. While this may hold true in certain industries like car manufacturing, it falls short in the context of apps due to two key factors. First, a realistic value for K would necessitate a large dimensionality, considering all the things we can do with superapps like WeChat. Second, gathering even a subset of app characteristics relevant for competitive analysis is challenging. Therefore, I choose the product-space approach and refrain from parameterizing $\gamma_{jj'}$. Instead of analyzing 100 apps in one model, which involves a gigantic matrix $\mathbf{\Gamma}$, I analyze a smaller model with more assumptions and only two apps of interest. Two is certainly not an ideal number. However, mergers in the mobile Internet industry are mostly about two apps (for example, Facebook's acquisition of Instagram). Ultimately, the number of apps researchers can analyze is constrained by computational limitations and the availability of overlapping user data for relevant app pairings.[9]

[9] One may also be concerned about the number of exclusion restrictions. This is not an issue if we use updates as IV. The update history of each app will be interacted with residuals of all other apps and *vice versa*. Therefore, the number of exclusion restrictions increases quadratically as well.

2.2 A Simplified Model

In the model to be estimated, there are four options $j = 0, 1, 2, 3$. $j = 1, 2$ are the two apps of interest. $j = 0$ is the option of not using a smartphone and $j = 3$ is a generic app which is to use any other apps. The utility maximization problem of consumer i in market $m = 1, 2, ..., M$ is

$$\max_{t_{i0m}, t_{i1m}, t_{i2m}, t_{i3m} \geq 0} t_{i0m} - 0.0005 t_{i0m}^2 + \sum_{j=1}^{2} \mu_{ijm} t_{ijm} + 2 t_{i3m} + \frac{1}{2} \sum_{j=1}^{3} \gamma_{ijm} t_{ijm}^2 + \gamma_{12} t_{i1m} t_{i2m} \tag{2}$$

$$s.t. \quad t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = 168$$

I add more assumptions compared to (1). To normalize the level of the utility function, I assume $\mu_{i0m} = 1$. I assume $\mu_{i3m} = 2$ so that all users spend a positive amount of time on the generic app.[10] μ_{i3m} can be any number greater than μ_{i0m}. Because the time spent on $j = 0$ is a residual term ($t_0 = 168 - t_1 - t_2 - t_3$) in the model, I assume γ_{i0m} to be a non-positive constant -0.001 ($\frac{1}{2} \times 0.001 = 0.0005$). I also assume $\gamma_{10} = \gamma_{20} = \gamma_{13} = \gamma_{23} = 0$ because those who use either app 1 or app 2 will always use the generic app and spend some time on offline activities and $(\gamma_{10}, \gamma_{20}, \gamma_{13}, \gamma_{23})$ cannot be identified. All the simplifying assumptions are made because I do not have additional variations.[11]

168 is the total number of hours in a week and the time scope of this

[10] All users in my dataset spend a positive amount of time on Android smartphones. Otherwise they are not observed.

[11] See Table 4.

utility function. This choice is imposed by the data structure; I happen to observe weekly usage. One can certainly consider utility functions defined over various time periods like a month, a day, an hour, or even a second if the data allows. Estimated demand models will be different but valid within their respective time scopes. For instance, when modeling usage for every second, all apps are substitutes. Depending on the question of interest, we might opt for different time scopes. For instance, if we want to study the effects of marketing campaigns, we want to set T to be day or week rather than year. If we have perfect data, we should choose T spanned by the observed usage of all options of interest. If users never use smartphones between 11 pm and 7 am and we are interested in app usage, then we should consider $T = 16$ for daily data or $T = 16 \times 7 = 112$ for weekly data. As I will show in the discussion of Netflix in section 7.3, different choices of T can reveal different competition patterns in different time scopes.

2.3 Consumer Heterogeneity

Consumers have different preferences for apps. μ_{ijm} and γ_{ijm} are parameterized as

$$\mu_{i1m} = \mathbf{x}_m \boldsymbol{\beta}_1^{\mu} + \xi_{1m}^{\mu} + \varepsilon_{i1m} = \delta_{1m}^{\mu} + \varepsilon_{i1m} \tag{3}$$

$$\mu_{i2m} = \mathbf{x}_m \boldsymbol{\beta}_2^{\mu} + \xi_{2m}^{\mu} + \varepsilon_{i2m} = \delta_{2m}^{\mu} + \varepsilon_{i2m} \tag{4}$$

$$\gamma_{i1m} = \mathbf{x}_m \boldsymbol{\beta}_1^{\gamma} + \xi_{1m}^{\gamma} = \delta_{1m}^{\gamma} \tag{5}$$

$$\gamma_{i2m} = \mathbf{x}_m \boldsymbol{\beta}_2^{\gamma} + \xi_{2m}^{\gamma} = \delta_{2m}^{\gamma} \tag{6}$$

$$\gamma_{i3m} = \mathbf{x}_m \boldsymbol{\beta}_3^{\gamma} + \xi_{3m}^{\gamma} = \delta_{3m}^{\gamma} \tag{7}$$

where $\mathbf{x}_m$ is a set of exogenous market level variables. I follow Berry *et al.* (1995) and Nevo (1998) in denoting market-level parameters with $\boldsymbol{\delta} = (\delta_{1m}^{\mu}, \delta_{2m}^{\mu}, \delta_{1m}^{\gamma}, \delta_{2m}^{\gamma}, \delta_{3m}^{\gamma})$. ξ^{μ} and ξ^{γ} capture app-market specific idiosyncratic error terms. For example, a weather shock to market m may increase the marginal utility of Uber but not that of Google Docs. ε_{i1m} and ε_{i2m} are individual error terms that are iid across individuals but can be correlated across apps. ε_{i1m} and ε_{i2m} capture unobserved individual characteristics that influence utilities derived from apps. For example, users with cars, compared to those without cars, derive higher utilities from Google Maps and lower utilities from Uber. Therefore, the preference for Uber and the preference for Google Map can be negatively correlated. As discussed in Train (2009), the variance of μ_{ijm} cannot be separately identified from the mean of μ_{ijm}. I assume $(\varepsilon_{i1m}, \varepsilon_{i2m})$ follows a normal distribution $N(\mathbf{0}, \mathbf{\Sigma})$[12], where

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

ρ captures correlated preferences. As we add more controls in $\mathbf{x}_m$, ρ may be closer to zero. Given that we can never control for all relevant factors at the individual level, we should not assume $\rho = 0$. γ_{12} and ρ together explains the overlapping user between app 1 and app 2.[13] An econometric challenge is to disentangle γ_{12} from ρ, which will be addressed in Section §5.

3 Budget Competition and Functional Competition

This section formally defines and distinguishes budget competition from functional competition. I then provide an analytical characterization of budget competition in the quadratic utility framework. This characterization reveals that budget competition can dominate functional competition and a pair of complementary apps can be gross substitutes. Lastly, I propose a descriptive index to gauge budget competition. The index highlights that budget competition between two apps grows with the correlation in their usage and increase quadratically with time spent on the apps. This index requires minimal data and can be used by industry analysts and antitrust authorities to quickly assess

[12] Any reasonable distribution would be compatible with my model.

[13] One can certainly add individual error terms in γ_{i1m} and γ_{i2m} as well and allow them to be correlated. However, I do not have variations to identify this correlation parameter.

competition between apps.

3.1 Definition

When an app is shut down,[14] users will reallocate their time to the remaining apps. There are two reasons the exit of an app could affect other apps. First, because of substitutability (complementarity), users find the remaining apps more (less) appealing. Hence users will spend more (less) time on the remaining apps. This is the functional competition effect. Second, the exit of an app means time that used to be spent on that app is now "free", and users can allocate it to the remaining apps. This is the budget competition effect.

Consider the original bundle, $\mathbf{t}^o = \arg\max U(\mathbf{t})$, and the final bundle, $\mathbf{t}^f = \arg\max U(\mathbf{t}|\mu_j = -\infty)$, subject to the same time constraint $\sum_{k=0}^{J} t_k = T$. $\mathbf{t}^f - \mathbf{t}^o$ summarizes the effects of the exit of app j. To formally separate budget competition and functional competition, I introduce an intermediate step. In the intermediate step, the user chooses an intermediate bundle, $\mathbf{t}^i$, such that the marginal utilities of $\mathbf{t}^i$ equal to the marginal utilities of $\mathbf{t}^o$ except for app j. That is, $\mathbf{t}^i$ is the solution to the following system of equations:

$$\frac{\partial U(\mathbf{t}^i|\mu_j = -\infty)}{\partial t_k^i} = \frac{\partial U(\mathbf{t}^o)}{\partial t_k^o} \quad \forall k \neq j \& t_k^i \geq 0 \tag{8}$$

$$t_j^i = 0.$$

Note that the time constraint is irrelevant in this step. $\mathbf{t}^i - \mathbf{t}^o$ is the functional

[14] The following analysis applies to entry and price changes as well.

competition effect because the difference is entirely due to complementarity or substitutability among apps. $\mathbf{t}^f - \mathbf{t}^i$ is therefore the budget competition effect. In the context of quadratic utility functions, we have a system of linear equations. If we have estimates of the parameters in the previous section, budget competition and functional competition are easy to calculate.

The above definition is applicable to entry and price changes as well. Consider pay-per-use by adding $-\alpha \sum_{j=0}^{J} p_j t_j$ into $U(t)$ where p_j is the hourly price of app j. The "net" taste parameter is $\mu_j - \alpha p_j$. A change in price corresponds to a change in taste parameter. When p_j changes from p^o to p^f, the intermediate bundle is given by

$$\frac{\partial U(\mathbf{t}^i | p_j = p^f)}{\partial t_k^i} = \frac{\partial U(\mathbf{t}^o | p_j = p^o)}{\partial t_k^o}.$$

Consumers face multiple constraints (notably, time and money). Budget competition works with multiple constraints. The intermediate bundle $\mathbf{t}^i$ in (8) is not the outcome of an optimization and can always be calculated regardless of the number of constraints. For most products, the monetary constraint is the only relevant constraint. Housing and food expenditure would crowd out discretionary expenses. Purchasing a new car would likely reduce vacation expenditures. Nevertheless, previous literature has not formally explored budget competition. The key reason is that the budget shares of traditional products like cars, cereals, and yogurt are generally small. In contrast, the time shares of leading apps can be large. A user may spend two hours on WeChat and two hours on Tik Tok within a single day. As we will see later, budget competition increase quadratically with budget shares.

3.2 Relationship to Slutsky Decomposition

The concepts of budget competition and functional competition relates to the substitution effect and the wealth effect in the classical demand theory. With Slutsky decomposition, we isolate the wealth effect of a price change through wealth compensation, ensuring that the original bundle is just affordable or the original level of utility is just attainable (Mas-Colell *et al.*, 1995). In this paper, I isolate the budget competition effect of a price change by restoring the marginal utilities to the original level. In Table 1, I compare the two ways of decomposition and the two sets of definitions of complements and substitutes considering the effects of an increase in $p_{j'}$ on product j. Whereas the wealth effect can be negative (for normal goods) or positive (for inferior goods), the budget competition effect always remains positive. A notable implication from Slutsky decomposition is that when the wealth effect dominates the substitution effect, the Walrasian demand for an inferior good increases after an increase in its own price. A similar surprise arises from my decomposition: when budget competition dominates functional competition, complementary goods are gross substitutes. This idea will be formally elucidated in the subsequent subsection.

Table 1: The Effects of An Increase in $p_{j'}$ on Product j

Product Relationship	Definition	Decomposition	
		Substitution Effect	Wealth Effect
Substitutes	$\frac{\partial h_j(p,u)}{\partial p_{j'}} > 0$	+	- (normal); + (inferior)
Complements	$\frac{\partial h_j(p,u)}{\partial p_{j'}} < 0$	-	
		Functional Competition	Budget Competition
Substitutes	$\frac{\partial^2 U(x)}{\partial x_j \partial x_{j'}} < 0$	+	+
Complements	$\frac{\partial^2 U(x)}{\partial x_j \partial x_{j'}} > 0$	-	+

Note:
1. $h_j(p,u)$ is the Hicksian demand of product j given prices p and a utility level u. $U(x)$ is the utility of consuming x.
2. The upper panel presents Slutsky decomposition and the lower panel my decomposition.

3.3 Analytical Characterization

In the model to be estimated, (8) can be simplified to one single equation when app 2 exits the market[15]:

$$\mu_1 + \gamma_1 t_1^o + \gamma_{12} t_2^o = \mu_1 + \gamma_1 t_1^i.$$

In Table 2, I provide general analytical solutions of functional competition and budget competition depending on if t_1^o and t_1^i are strictly positive. Note that for app 2 to have any competitive effect, be it budget competition or functional competition, t_2^o must be strictly positive, which is implicitly assumed in Table 2. The results in Table 2 are intuitive. Let us focus on the first row and assume $\gamma_{12} \leq 0$, $t_1^o > 0$ and $t_1^i > 0$. In the intermediate step, $\frac{\gamma_{12}}{\gamma_1} t_2^o$ are diverted

[15]The marginal utilities of app 3 and the offline option would not change with the exit of app 2 because I assume $\gamma_{20} = \gamma_{23} = 0$.

to app 1 due to functional competition. That leaves the user with $(1-\frac{\gamma_{12}}{\gamma_1})t_2^o$ of free time, which is allocated to the remaining options *proportional to the inverse of their satiation parameters.* The intuition is similar for complements. When $\gamma_{12} > 0$, t_1^i decreases by $|\frac{\gamma_{12}}{\gamma_1}|t_2^o$. Therefore, the free time is $(1+|\frac{\gamma_{12}}{\gamma_1}|)t_2^o$ and larger than t_2^o. For users with $t_1^o > 0$ and $t_1^i > 0$, the gross diversion ratio consists of two parts: diversion because of functional competition ($\frac{\gamma_{12}}{\gamma_1}$) and diversion because of budget competition ($(1-\frac{\gamma_{12}}{\gamma_1})\frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3}}$):

$$\text{Diversion Ratio} \equiv \frac{t_1^f - t_1^o}{t_2^o} = \frac{\gamma_{12}}{\gamma_1} + (1-\frac{\gamma_{12}}{\gamma_1})\frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3}}. \tag{9}$$

From Table 2 and (9), budget competition is increasing in γ_{12} and γ_1. An important implication of (9) is that budget competition can dominate functional competition and apps with a positive γ_{12} can be gross substitutes. How likely is this scenario? The answer depends on $\frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3}}$, which describes how free time is allocated to remaining options. It is a nonlinear function of deep parameters in a structural model. To calculate the size of budget competition, we must estimate a full model as in Section §6. However, with some assumptions, we can transform $\frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3}}$ into ratios of real-world variables. (9) predicts cross-category substitution (positive diversion ratio when $\gamma_{12} = 0$) when an app is shut down. This prediction is confirmed by the experimental results in Aridor (2023). When Instagram[16] was disabled, time spent on communication apps and entertainment apps increased.

[16]Instagram is in the social category.

Table 2: Analytical Decomposition

t_1^o	t_1^i	Functional Competition $(t_1^i - t_1^o)$	Budget Competition $(t_1^f - t_1^i)$
$t_1^o > 0$	$t_1^i > 0$	$\frac{\gamma_{12}}{\gamma_1} t_2^o$	$(1 - \frac{\gamma_{12}}{\gamma_1}) t_2^o \frac{1}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}$
$t_1^o > 0$	$t_1^i = 0$	$-t_1^0$	$\max\{0, \frac{T - \frac{\mu_1 - \mu_0}{\gamma_0} - \frac{\mu_1 - \mu_3}{\gamma_3}}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}\}$
$t_1^o = 0$	$t_1^i > 0$	$\frac{\mu_2 - \mu_1}{\gamma_1} + \frac{\gamma_2}{\gamma_1} t_2^o$	$[(1 - \frac{\gamma_2}{\gamma_1}) t_2^o - \frac{\mu_2 - \mu_1}{\gamma_1}] \frac{1}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}$
$t_1^o = 0$	$t_1^i = 0$	0	$\max\{0, \frac{T - \frac{\mu_1 - \mu 0}{\gamma_0} - \frac{\mu_1 - \mu_3}{\gamma_3}}{\gamma_1(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3})}\}$

Note: This table presents analytical characterizations of functional competition and budget competition. The calculations are in Appendix B.

3.4 A Descriptive Index

I propose a descriptive index to gauge budget competition. To motivate this index, consider two apps of interest, app 1 and app 2. Suppose we observe a user and her time spent on app 1 and app 2, t_1^* and t_2^*. The utility function of this user is

$$\max_{t_0, t_1, t_2 \geq 0} t_0 + t_1 + t_2 - 0.0005 t_0^2 + \frac{1}{2}\gamma_1 t_1^2 + \frac{1}{2}\gamma_2 t_2^2 \tag{10}$$
$$s.t. \quad t_0 + t_1 + t_2 = T$$

where t_0 is the time spent on any other activities, online or offline. I assume $\mu_1 = \mu_2 = 1 = \mu_0$ and $\gamma_{12} = 0$ to avoid complicated estimation and obtain a closed-from index. The solution to this maximization problem is $(t_0, t_1, t_2) = (T - t_1^* - t_2^*, t_1^*, t_2^*)$. We can solve for γ_1 and γ_2 with the FOCs. Consider the exit of app 2:

$$\max_{t_0,t_1\geq 0} t_0 + t_1 - 0.0005t_0^2 + \frac{1}{2}\gamma_1 t_1^2 \tag{11}$$
$$s.t. \quad t_0 + t_1 = T$$

Let $(T - t_1', t_1')$ be the optimal time allocation when app 2 is absent. I prove in the appendix that

$$\Delta t_1 = t_1' - t_1^* = t_2^*(\frac{t_1^*}{t_0^* + t_1^*}) = t_2^*(\frac{t_1^*}{T - t_2^*}). \tag{12}$$

When app 2 exits the market, time that used to be spent on app 2 will be reallocated to the remaining apps *proportional to their time shares before the exit.*[17] $\frac{\frac{1}{\gamma_1}}{(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3})}$ in our previous analysis becomes $\frac{t_1^*}{t_0^*+t_1^*} = \frac{t_1^*}{T-t_2^*}$.[18] The budget competition effect of app 2 on app 1 is increasing in t_1^* and t_2^*. The numerator in (12) are the same for app 1 and app 2. Therefore, budget competition is largely symmetric between app 1 and app 2 unless $T - t_1 \gg T - t_2$ or the reverse. Adding more independent options (apps) would not change (12).

If we observe aggregate usage of one single market, we can adopt a representative user approach and plug in average usage of app 1 and app 2 into (12). When we observe a group of users, the overall budget competition effect

[17] This is similar to the prediction from a simple logit model: when a product exits the market, its market share will be reallocated to the remaining products *proportional to their market shares before the exit.*

[18] t_3 merges into the outside option.

is

$$\sum_i \Delta t_{i1} = \sum_i t^*_{i2} \left(\frac{t^*_{i1}}{T - t^*_{i2}} \right). \tag{13}$$

The key takeaway from (12) and 13 is that budget competition grows with the correlation between t^*_{i1} and t^*_{i2} and increases **quadratically** with time spent on the apps. Consequently, mergers involving apps with significant time shares or apps targeting the same niche market should trigger scrutiny from antitrust authorities, irrespective of their functional aspects. Candidates are superapps (WeChat, Alipay, Baidu), gaming apps (PUBG, Honor of Kings), and video and streaming apps (YouTube, Tik Tok, Twitch, Kwai, Bilibili, Netflix).

Two assumptions are crucial to this index. First, I assume all options to be independent to avoid estimating γ_{12} in a full model. Therefore this index is more accurate when we expect $\frac{\gamma_{12}}{\gamma_1}$ to be close to zero. However, we can combine our belief of γ_{12} with the index using (9). Second, I assume $\mu_1 = \mu_2 = 1 = \mu_0$ to have a closed-form index of budget competition. Therefore, any difference in t_1 and t_2 is attributed to γ_1 and γ_2. This would be a serious concern if $s_1 \gg s_2$ or $s_1 \ll s_2$. In this case, we can focus on overlapping users of app 1 and app 2 and $s_1 = s_2 = 1$ in this market. In general, we can segment the market and apply the index to sub-markets where s_1 and s_2 are similar and close to 1 or using a representative user approach. After estimating the full model, we can validate the index by comparing the index with results from the full model in Section §7.

4 Data

There are two types of app usage data available in the mobile Internet industry: individual level data and market level data. The first type of data resembles traditional surveys: firms pay individuals for their permission to install an app or software in order to monitor the usage of their devices. The data sets used by Han *et al.* (2016) , Lee (2018), Wu *et al.* (2022), and Boik *et al.* (2016) fall into this category. The data set used in this paper is aggregate market data, which is estimated based on a large quantity of observations from different sources. Wireless carriers and app developers are the two major sources. For instance, China Unicom provides app usage data based on traffic data from its users.[19] App developers mostly use third-party libraries to analyze behaviors of their users.[20] Those data are then traded and matched based on unique device identifiers. In sum, market level data are estimated from snapshots of millions of devices, whereas individual data are 24×7 information from thousands of users. While both individual level data and market level data can be used to estimate relationships among apps, market level data are widely available in different countries and raise much less privacy concerns.[21]

Data used in this paper are from iResearch, a leading consulting firm in China with a focus on the mobile Internet industry. There are three parts of our data: the app usage data, the smartphone usage data, and the overlapping

[19] See https://www.cubigdata.cn

[20] For a story about how this works, see the report by the Wall Street Journal: https://www.wsj.com/articles/you-give-apps-sensitive-personal-information-then-they-tell-facebook-11550851636?mod=article_inline

[21] For example, Facebook shut down its “Facebook Research” app because of public anger. See https://www.wired.com/story/facebook-research-app-root-certificate

user data. I introduce these data in the following subsections. All data are weekly data taken from the first quarter of 2017 in China.

4.1 App Usage Data

I acquired app usage data of the top 300 apps on Android cellphones of 290 demographic groups for 13 weeks in China. In this data set, a market is a demographic group defined by gender (male and female), age groups (below 24, 25-30, 31-35, 36-40, and above 40), and geographic areas (28 provinces and an “other” category). I do not have all the 300 apps’ data as some apps have an estimated number of active users that is too small to be reliable. The threshold is 50,000. On average, I observe about 82 apps for each week-market pair. I observe more apps for large demographic groups in the data set. In total, I have 312,724 week-market-app observations. For each unit of observation, I observe the number of devices (per ten thousands) that used the app at least once during the week (henceforth, active user) and the average number of minutes spent on the app per device during the week (henceforth, average time spent). The summary statistics are in the upper panel of Table 3. The zeros in the table result from the technical difficulty in estimating usage of some apps (for example, input methods).

4.2 Smartphone Usage Data

iResearch provides total usage of Android devices, i.e., the smartphone usage data. Similarly, I have the number of active devices (per ten thousands) that

Table 3: Summary Statistics of Spp Usage

Variables	Mean	Min	Max	StdDev	# Obs	Unit
App Usage Data						
Active user	28.05	5	1074.2	46.38	312724	ten thousands
Market share	0.1064	0.004	0.958	0.132	312724	-
Average time spent	58.65	0	800.17	74.86	312724	minutes
Smartphone Usage Data						
Active user	225.05	10.31	1238.75	193.92	3770	ten thousands
Average time spent	1006	561.5	1435.5	202.08	3770	minutes
Overlapping User Data						
Overlapping user	599.21	6.41	29979.13	1387.733	79809	ten thousands

Note:
1, The smartphone and app usage data are weekly observations at the demographic group level from the first 13 weeks of 2017 in China. The overlapping user data are weekly aggregate data for each pair of apps.
2. Active user of an app is the number of devices that used the app at least once during the week. Active user of smartphone is the number of Android smartphones that are used at least once during the week. Average time spent is the average number of minutes spent on the app per device during the week. Market share of an app is the active user of this app divided by the active user of Android smartphones in that market. Overlapping user is the number of Android smartphones that use both apps at least once during the week.
3, The zeros in app usage data result from the technical difficulty of estimating usage of some apps, for example, input methods.
Data Source: iResearch.

are used at least once during the week (active users) and the average number of minutes spent on Android smartphones per device during the week (average time spent). With those data, I calculate market shares of apps in each market which is the number of active users of an app divided by the number of active users of Android smartphones in that market. The summary statistics are in the middle panel of Table 3.

4.3 Overlapping User Data

Most importantly, I have the overlapping user data. For each pair of apps, I observe the number of Android smartphone users who used both apps at least once during the week (henceforth, overlapping users). Again the 50,000 threshold applies. On average, I observe about 110 apps each week. I only have overlapping user data at the national level because they are small, and hence unreliable, at the demographic group level. The summary statistics are in the lower panel of Table 3.

5 Estimation

5.1 Notation

I use GMM to match moments predicted by the model with moments calculated from the data. The full set of parameters to be estimated are $\boldsymbol{\theta} = (\boldsymbol{\beta}_1^{\mu}, \boldsymbol{\beta}_2^{\mu}, \boldsymbol{\beta}_1^{\gamma}, \boldsymbol{\beta}_2^{\gamma}, \boldsymbol{\beta}_3^{\gamma}, \gamma_{12}, \rho)$. As in Nevo (1998), denote the linear parameters with $\boldsymbol{\theta}_1 = (\boldsymbol{\beta}_1^{\mu}, \boldsymbol{\beta}_2^{\mu}, \boldsymbol{\beta}_1^{\gamma}, \boldsymbol{\beta}_2^{\gamma}, \boldsymbol{\beta}_3^{\gamma})$ as they will enter the GMM function linearly and the nonlinear parameters with $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$. I observe a set of markets, which are defined to be demographic groups, for 13 weeks. Denote weeks with w. For each market-week unit, I observe t^*_{1mw}, t^*_{2mw} and t^*_{3mw}, the average time spent on app 1, app 2, and all other apps in hours. For each week, I also observe the total number of overlapping user between app 1 and app 2, c^*_{12w}. The asterisks indicate that they are observed variables. Hence the endogenous variables to be explained are $\mathbf{y}^*_{mw} = (s^*_{1mw}, s^*_{2mw}, t^*_{1mw}, t^*_{2mw}, t^*_{3mw})$ and c^*_{12w}.

The exogenous variables include $\mathbf{x}_{mw}$, a set of week and market fixed effects. Note that $\boldsymbol{\delta} = \mathbf{x}_{mw}\boldsymbol{\beta} + \boldsymbol{\xi}$.

With those notations, the model can be succinctly summarized as

$$(\mathbf{y}^*_{mw}, c^*_{12w}) = f(\boldsymbol{\delta}, \gamma_{12}, \rho) = f(\mathbf{x}_{mw}\boldsymbol{\beta} + \boldsymbol{\xi}, \gamma_{12}, \rho)$$

where $f(\cdot)$ is the nonlinear model described in Section §2 and $\boldsymbol{\xi}$ is the stack of all market level error terms. Note that there are five components in $\boldsymbol{y}^*_{mw}$ and five components in $\boldsymbol{\delta}$. At the market level, we have six outcome variables but seven parameters. The model is not identified with the observed variables we have.

5.2 Identification

The econometric challenge is to identify γ_{12} from ρ. Intuitively, both γ_{12} and ρ can explain c^*_{12w}. If one observes that many users use both NYTimes and WSJ, it could be the case that NYTimes and WSJ are complements as they offer different perspectives on the same events, or that users have a strong demand of news in general. In the first case, $\gamma_{12} > 0$. In the second case, $\rho > 0$. In economic textbooks, complements and substitutes are defined with compensated cross-price elasticities of demand: if an exogenous increase in the price of product A leads to a decrease in the compensated demand of product B, then they are complements; otherwise, they are substitutes. When there is no price, one can extend the definition: if users spend more time on an app due to an exogenous increase in its utility, the (marginal) utility of its complements (substitutes) would increase (decrease). This definition, based on cross-derivatives of the utility functions, forms the foundation of my identification strategy, which utilizes app updates as instrumental variables. Updates of app 1 should change the *utility* of app 1 but not that of app 2. However, updates of app 1 can change the *usage* of app 2 through γ_{12}. Therefore, I use the following moments to identify nonlinear parameters γ_{12} and ρ

$$E(c^*_{12} - c_{12}) = 0 \tag{14}$$

$$E(update_{2w} \cdot \xi^{\mu}_{1mw}) = 0 \tag{15}$$

$$E(update_{1w} \cdot \xi^{\mu}_{2mw}) = 0 \tag{16}$$

The moment in (14) matches the observed overlapping user and the predicted overlapping user given γ_{12} and ρ. The moments in (15) and (16) are based on the assumption that the update history of app 1 (app 2) should not enter the utility of app 2 (app 1) directly. As Android update data is unreliable,[22] I utilize the update history of the corresponding iOS app, which cannot affect the utility of any other Android app. Specifically, update history is described by three variables: the cumulative numbers of small updates, medium updates, and major updates.[23] Using cumulative values allows my IVs to capture update effects even if adoption is not immediate. As indicated by the subscript in $update_{1w}$, this history is common across all users within China and collinear with time fixed effects. To address this problem, I construct

[22] One reason is that developers can publish Android apps outside mainstream app stores.

[23] "Small", "medium", and "major" are defined by the digits of version numbers.

market-specific update history variables, enabling each market to respond to updates differently. Therefore, there are at most $3 \times M$ moments implied by (15). In Section §A, I provide reduced form evidence that the overlapping user data and the update history contain new information about the relationship between apps beyond the correlation of active users.

The identification of linear parameters $\boldsymbol{\beta}$ is straightforward and relies on the following moment conditions:

$$E(\mathbf{x}'_{mw}\xi^{\mu}_{1mw}) = 0 \tag{17}$$

$$E(\mathbf{x}'_{mw}\xi^{\mu}_{2mw}) = 0 \tag{18}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{1mw}) = 0 \tag{19}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{2mw}) = 0 \tag{20}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{3mw}) = 0 \tag{21}$$

Overall, the identifying variations for each parameter is listed in Table 4. Table 4 highlights the justification for the simplifying assumptions made in section 2.2 and section 2.3: I do not have additional variations to estimate other parameters.

5.3 Implementation

Based on the above moments from (14) to (21), the GMM estimation is to minimize

$$\min_{\boldsymbol{\theta}} \boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi} + (c^*_{12} - c_{12})^2 \tag{22}$$

Table 4: Identifying Variations

Parameters	Variables	Data
$\boldsymbol{\beta}_1^{\mu}$	s_{1mw} $(\frac{1}{N_{mw}}\sum_i t_{i1mw} > 0)$	Active users of app 1
$\boldsymbol{\beta}_2^{\mu}$	$s_{2mw}(\frac{1}{N_{mw}}\sum_i t_{i2mw} > 0)$	Active users of app 2
$\boldsymbol{\beta}_1^{\gamma}$	$t_{1mw}(\frac{1}{N_{mw}}\sum_i t_{i1mw})$	Average time spent on app 1
$\boldsymbol{\beta}_2^{\gamma}$	$t_{2mw}(\frac{1}{N_{mw}}\sum_i t_{i2mw})$	Average time spent on app 2
$\boldsymbol{\beta}_3^{\gamma}$	$t_{3mw}(\frac{1}{N_{mw}}\sum_i t_{i3mw})$	Average time spent on Smartphone
γ_{12}, ρ	c_{12}, $update_{1w}$, $update_{2w}$	Overlapping user and updates

Note: w denotes the index for weeks, while m represents the index for markets. N_{mw} stands for the number of smartphone users in market m during week w.

where $\boldsymbol{\xi}$ is the stack of all market level error terms and $\mathbf{z}_{mw} = (\mathbf{x}_{mw}, updata_{1w}, updata_{2w})$ collects all the exogenous variables. I separate $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ from $(c^*_{12w} - c_{12w})^2$ to highlight the fact that $\boldsymbol{\theta}_1$ enters $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ linearly and does not enter $(c^*_{12w} - c_{12w})^2$ given $\boldsymbol{\delta}$. Therefore, we can limit the global search to $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$ as $\boldsymbol{\theta}_1$ is a linear function of $\boldsymbol{\delta}$.

This estimation follows Berry *et al.* (1995) with an inversion step and a global search step. I need to find the values of $\boldsymbol{\delta}$ that match the five observed market outcomes $\mathbf{y}^*_{mw} = (s^*_{1mw}, s^*_{2mw}, t^*_{1mw}, t^*_{2mw}, t^*_{3mw})$ given (γ_{12}, ρ). This is to solve the following system of nonlinear equations,

$$\mathbf{y}^*_{mw} = \mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho). \tag{23}$$

Note that each component in $\mathbf{y}_{mw}$ is monotonically increasing in the corresponding component in $\boldsymbol{\delta}$. For example, given $(\delta^{\mu}_{2mw}, \delta^{\gamma}_{1mw}, \delta^{\gamma}_{2mw}, \delta^{\gamma}_{3mw})$ and (γ_{12}, ρ), s_{1mw} is increasing in δ^{μ}_{1mw}. I solve (23) by iterating on $\boldsymbol{\delta}$ analogously to the contraction mapping used by Berry *et al.* (1995) and Gowrisankaran &

Rysman (2012):

$$\boldsymbol{\delta}^{new} = \boldsymbol{\delta}^{old} + \boldsymbol{\phi} \cdot \{ln(\mathbf{y}^*_{mw}) - ln(\mathbf{y}_{mw}(\boldsymbol{\delta}^{old}, \gamma_{12}, \rho))\} \tag{24}$$

where $\boldsymbol{\phi}$ are five positive tuning parameter used in the iterations.

Despite the appealing features of quadratic utility functions, there is no analytical solution to quadratic optimization problems. Therefore, I use numerical integration to form expectations of $\mathbf{y}_{mw}$. Let N_s be the number of simulations used for integration. We have

$$\mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho) = \frac{1}{N_s} \sum_{n=1}^{N_s} \mathbf{y}_{nmw}(\boldsymbol{\delta}, \gamma_{12}) \tag{25}$$

where $\mathbf{y}_{nmw}$ are the individual outcome for the nth draw of $(\varepsilon_1, \varepsilon_2)$. In practice, I use 1000 Halton draws in the integration.

To summarize, the estimation consists of the following steps:

1. For a pair of (γ_{12}, ρ), invert out $\boldsymbol{\delta}(\gamma_{12}, \rho)$ with the mapping described in (24).

2. Calculate $c_{12}(\boldsymbol{\delta}(\gamma_{12}, \rho), \gamma_{12}, \rho)$ and $\boldsymbol{\xi}(\boldsymbol{\delta}(\gamma_{12}, \rho), \mathbf{z})$ and the value of GMM function in (22).

3. Find (γ_{12}, ρ) that minimizes the GMM value calculated in step 2.

6 Estimation Results

I estimate the model on three representative pairs of apps to see how the model performs in different scenarios. For the first two pairs, I choose them because they are obviously a pair of substitutes (Baidu Map and Amap) and a pair of complements (Baidu and Baidu Map). A satisfactory model can infer the relationships from data. I study WeChat and Kwai because users spend a lot of time on them so that budget competition may be salient and the relationship between the two is *a priori* ambiguous. To reduce the computation burden, I aggregate market outcomes over provinces.[24] Therefore, for each pair of apps, I have a panel of 11 markets[25] for 13 weeks.

6.1 Substitutes

The first pair of apps are Baidu Map (app 1) and Amap (app 2), two dominant players in China's mobile map market. Over the 13-week observation period, Baidu Map's active user base grew from 90 million to 110 million, while Amap's user base increased from 75 million to 100 million. Notably, the number of users using both Baidu Map and Amap rose from 11 million to 18 million. The summary statistics of market level variables are in Table 5.

The first three columns of Table 6 present the estimates of γ_{12} and ρ using different IVs. These estimates exhibit consistent signs and comparable magni-

[24] In an earlier version of this paper, I estimate the model without such aggregation. The estimated competition patterns are similar to the results reported here. That version of paper is available upon request.

[25] Gender and five age groups define 10 markets; and an "other" market to account for the difference between national usage and the total usage of the balanced market panel.

Table 5: Summary Statistics of Baidu Map and Amap

Variables	Mean	StdDev	Min	Max	Unit
$s^*_{BaiduMap}$	0.1463	0.032	0.0877	0.2414	-
s^*_{Amap}	0.1277	0.0286	0.0832	0.2164	-
$t^*_{BaiduMap}$	0.0367	0.0085	0.0195	0.0652	hour
t^*_{Amap}	0.0746	0.0192	0.0421	0.1564	hour
t^*_{3mw}	16.6915	3.1601	10.5834	21.8704	hour

Note: $s^*_{BaiduMap}$ (s^*_{Amap}) is the number of active users of Baidu Map (Amap) divided by the number of active users of Android cellphones. $t^*_{BaiduMap}$ (t^*_{Amap}, t^*_{3mw}) is the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphone.
Data Source: iResearch.

tudes. I use column (3) as my main results because both IVs are used. Baidu Map and Amap are estimated to be substitutes ($\hat{\gamma}_{12} = -1.15$), which confirms our prior belief. $\hat{\rho} = 0.7711$ suggests that Baidu map and Amap target the same group of users. Because the two apps offer similar functions, users who need Baidu Map will also find Amap useful. For the same reason, users who already use one would find the other redundant. A negative γ_{12} and a large ρ are characteristic of direct competitors. For comparison, I also estimate γ_{12} with the assumption $\rho = 0$[26] in column (4) of Table 6. In this specification, Baidu Map and Amap are estimated to be almost independent apps. γ_{12} and ρ "substitute" each other in explaining the overlapping user data: from column (3) to column (4), as ρ decreases from 0.7711 to 0, γ_{12} increases from -1.15 to -0.02.

γ_{12} is a structural parameter in a utility function. To understand the economic significance of γ_{12}, I quantify the value of substitutability/complementarity with compensating variation (CV). I calculate the compensating variations

[26] Without instruments, we do not have variations to estimate both γ_{12} and ρ.

Table 6: Estimates for Baidu Map and Amap

	(1)	(2)	(3)	(4)
γ_{12}	-0.95	-1.15	-1.15	-0.02
	(0.0009)	(0.0003)	(0.0003)	(0.0015)
ρ	0.592	0.7711	0.7711	0
	(0.0001)	(0.0004)	(0.0004)	-
$update_1$ as IV	No	Yes	Yes	No
$update_2$ as IV	Yes	No	Yes	No

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
Data Source: The author's calculations.

(CVs) of individual apps and pairs of apps. Specifically, the total time a user has is increased to compensate for the loss of an app (or the pair), ensuring that their maximized utilities remain identical before and after the hypothetical app (pair) shutdown. The difference between the sum of individual app CVs and the CV of the app pair captures the value of substitutability/complementarity. This utility specification aligns with the discrete model outlined in Gentzkow (2007), establishing that such discrete choice models are a specific case within the framework of this study. The CVs of Baidu Map, Amap, and the pair and the value of substitutability/complementarity are in Table 7. Substitutability is economically significant. As shown in the last column of Table 7., the CV of Baidu Map is 9.4 hours for 1000 smartphone users when Amap is available.[27] If Amap were to exit the market, consumers would have one fewer option, and the CV of Baidu Map would increase to $9.4 + 7.2 = 16.6$ hours.

[27] One might think 9.4 hours is a small number for an app like Baidu Map. Note that only 15% of the 1000 smartphone users use Baidu Map and the total time spent on Baidu Map for 1000 smartphone users is about 36.7 hours. Apart from substitutability/complementarity, the CV of Baidu Map is determined by the satiation parameters of the remaining options.

Table 7: Compensating Variations of Baidu Map and Amap

	(1)	(2)	(3)
CV of Baidu Map	10.7759	9.3945	9.3945
CV of Amap	18.6456	16.8407	16.8407
CV of Both	32.9073	33.4262	33.4262
Substitutability (Complementarity)	-3.4858	-7.1911	-7.1911
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-0.95,0.592)	(-1.15,0.7711)	(-1.15,0.7711)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The calculations are based on an anonymous market in the first week of 2017.
2, All numeric cells are the sum of CV in hours for all 1000 simulated smartphone users.
Data Source: The author's calculations.

6.2 Complements

The second pair of apps are Baidu (app 1) and Baidu Map (app 2), both developed by Baidu, Inc. The core functions of Baidu app are searching and news stream. I expect search engines and maps, like Baidu and Baidu Map, are complements. For example, when users search for locations, the first results often direct users to map apps. Throughout the 13-week observation period, Baidu's active user base fluctuated around 177 million, while the number of overlapping users between Baidu and Baidu Map increased from 30 million to 37 million. The summary statistics of market level variables are in Table 8. Note that there are slight differences between the summary statistics of s^*_{2mw} in Table 8 and the summary statistics of s^*_{1mw} in Table 5. This arises because the balanced panels used for the two pairs are slightly different.

The estimates of (γ_{12}, ρ) are in Table 9. The coefficients in the first column are quite different from those in the other two columns. This suggests that

Table 8: Summary Statistics of Baidu and Baidu Map

Variables	Mean	StdDev	Min	Max	Unit
s^*_{Baidu}	0.2494	0.0464	0.1555	0.3321	-
$s^*_{BaiduMap}$	0.146	0.0323	0.0876	0.2414	-
t^*_{Baidu}	0.3086	0.0524	0.1475	0.4001	hour
$t^*_{BaiduMap}$	0.0366	0.0085	0.0195	0.0652	hour
t^*_{3mw}	16.4493	3.1086	10.4568	21.5881	hour

Note: s^*_{Baidu} ($s^*_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t^*_{Baidu} ($t^*_{BaiduMap}$, t^*_{3mw}) is the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphone.
Data Source: iResearch.

the IV used in column (1) is not ideal. As before, I treat results in column (3) as the main results. The main results confirm our prior belief that Baidu and Baidu Map are complements. As in the previous subsection, I calculate the CVs of Baidu Map, Amap, and the pair and the value of substitutability/complementarity in Table 10. The CVs of Baidu Map are in line with those in Table 7. The complementarity is small in magnitude but large as a percentage of the CV of Baidu Map for columns (2) and (3). This suggests that Baidu Map relies on Baidu but not the reverse. Baidu has other apps named after Baidu and they are in the "Baidu Core" business unit. It is reasonable to assume that those apps are also complements with the Baidu app.

6.3 Independent Apps

The last pair of apps I studied are WeChat (app 1) and Kwai (app 2). WeChat, first realeased in 2011, is the flagship app of Tencent. By the first quarter of 2017, the main functions include instant messaging, social media ("Moments"),

Table 9: Estimates for Baidu and Baidu Map

	(1)	(2)	(3)
γ_{12}	-0.4613	0.296	0.1467
	(0.0006)	(0.0035)	(0.0005)
ρ	0.5522	-0.1642	-0.0448
	(0.0006)	(0.0001)	(0.0005)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
Data Source: The author's calculations.

mobile payment ("WeChat Pay"), content distribution ("Subscriptions"), and app store ("mini program"). It is a superapp used by almost all smartphone users in China. From Table 11, users spent about one fourth of their smartphone time on WeChat. Given it market dominance, WeChat is competing with all other apps for user time. Kwai is a video-sharing platform featuring short videos and live-streaming. Recommendation algorithms employed by Kwai and similar platforms have earned them the moniker "black holes of time" due to their capacity to engross users for extended periods. In terms of their functions, WeChat and Kwai seems to be independent or weak substitutes in the broad sense of social networking. However, a significant overlap exists in their user base. During the 13 weeks, the number of active users of WeChat fluctuated around 555 million, and those of Kwai increased from 78 million to 81 million. The number of overlapping users between WeChat and Kwai is about 70 million. It is tempting to conjecture that the two apps are complements based on the number of overlapping users. Overall, the compet-

Table 10: Compensating Variations of Baidu and Baidu Map

	(1)	(2)	(3)
CV of Baidu	189.5948	193.6434	194.2896
CV of Baidu Map	10.2916	10.8814	11.1509
CV of Both	210.6131	202.3202	204.2014
Substitutability (Complementarity)	-10.7266	2.2045	1.239
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-0.4613,0.5522)	(0.296,-0.1642)	(0.1467,-0.0448)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The calculations are based on an anonymous market in the first week of 2017.
2, All numeric cells are the sum of CV in hours for all 1000 simulated users.
Data Source: The author's calculations.

itive relationship between WeChat and Kwai is ambiguous.

Table 11: Summary Statistics of WeChat and Kwai

Variables	Mean	StdDev	Min	Max	Unit
s^*_{WeChat}	0.8352	0.0532	0.731	0.9346	-
s^*_{Kwai}	0.1154	0.0148	0.0864	0.1451	-
t^*_{WeChat}	4.3685	0.5473	3.2078	5.5869	hour
t^*_{Kwai}	0.1896	0.017	0.1519	0.2246	hour
t^*_{3mw}	12.2633	2.6741	6.7176	16.3154	hour

Note: s^*_{WeChat} (s^*_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t^*_{WeChat} (t^*_{Kwai}, t^*_{3mw}) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphone.
Data Source: iResearch.

The estimates of (γ_{12}, ρ) are in Table 12. Consistent with the previous analyses, the results in column (3) are considered the main results. $\hat{\gamma}_{12} = -0.08$ refutes the conjecture that WeChat and Kwai are complements. The large number of overlapping users is explained by the positive correlation observed between preferences for WeChat and Kwai ($\hat{\rho} = 0.42$). This suggests that the budget competition effect between the two apps may be significant. With these estimates, I calculate the CVs of WeChat, Kwai, and the pair and the value of substitutability/complementarity in Table 13. When WeChat is shut down, the CV of Kwai would increase by more than 150% in column (3). If Kwai is shut down, the CV of WeChat would also increase significantly. WeChat and Kwai are competing for user time despite a large number of overlapping users and seemingly independent functions. Note that this conclusion is based on data from 2017. After failed attempts to promote its own short-video app WeSee, Tencent invested \$2 billion in Kwai in December 2019[28], and is now the largest institutional investor of Kwai after its IPO. Tencent also added a short-video feature to WeChat in the first quarter of 2020 to directly compete with Kwai and Tik Tok.

Table 12: Estimates for WeChat and Kwai

	(1)	(2)	(3)
γ_{12}	-0.14	-0.02	-0.08
	(0.0001)	(0.0001)	(0.0015)
ρ	0.76	0.18	0.42
	(0.0001)	(0.0001)	(0.0024)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
Data Source: The author's calculations.

[28] See https://www.scmp.com/tech/apps-social/article/3041747/tencent-said-invest-us2-billion-short-video-app-kuaishou

Table 13: Compensating Variations of WeChat and Kwai

	(1)	(2)	(3)
CV of WeChat	3191.5925	3635.2672	3567.6392
CV of Kwai	70.0809	102.6322	94.9661
CV of Both	3853.5602	3765.7854	3817.5046
Substitutability (Complementarity)	-591.8868	-27.886	-154.8994
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-0.14,0.76)	(-0.02,0.18)	(-0.08,0.42)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The calculations are based on an anonymous market in the first week of 2017.
2, All numeric cells are the sum of CV in hours for all 1000 simulated users.
Data Source: The author's calculations.

6.4 Discussion

Comparing the results of the three pairs of apps, the substitutability/complementarity term is positive if and only if γ_{12} is positive. Estimated γ_{12} cannot be compared across models. $\hat{\gamma}_{12} = -1.15$ in column (3) of Tables 7 and $\hat{\gamma}_{12} = -0.08$ in column (3) of Table 13. By contrast, the substitutability/complementarity term is -7.2 hours in the first case and -154.9 hours in the second case.

In the above tables, I report only estimates of γ_{12} and ρ and coefficients of other covariates (fixed effects) are omitted because there are many fixed effects. In Section §B, I report week fixed effects and aggregate market fixed effects to demographic levels for Baidu Map and Amap. Male users and users aged between 31-35 derive higher utility from Baidu Map and Amap, due to a higher car ownership rate within those demographics in China.

The demand model can further incorporate other notable features in the mobile Internet industry (for example, advertisement and two-sidedness). In an earlier version of this paper, I estimate a similar model with network effects by adding active users of app 1 in a reference market in the marginal utility of app 1 in the focal market μ_{1mw} (Weiergraeber, 2022). I find strong network effects and the estimated competition patterns are similar.[29]

7 Budget Competition Revisited

In this section, I revisit budget competition with the estimated full model. To better understand the competitive relationship between apps, I shut down one of the two apps to see how the usage of the other app would change. This approach allows for the calculation of diversion ratios, which quantify the proportion of usage diverted from the exited app to the remaining one.[30] With these simulations, app developers would know who competes time away from their apps. I then decompose the competitive effects of one app on another into "functional competition" and "budget competition" according to the definition in Section §3. I compare the decomposition results with the descriptive index. Despite the simplifying assumptions of the index, the results are close. Lastly, I use several examples to discuss how we can combine the index with other institutional knowledge to more accurately assess budget competition.

[29]The results are available upon request.

[30]Diversion ratios are an important tool of antitrust authorities to analyze horizontal mergers. In the 2010 Horizontal Merger Guidelines, " Diversion ratios between products sold by one merging firm and products sold by the other merging firm can be very informative for assessing unilateral price effects, with higher diversion ratios indicating a greater likelihood of such effects."

7.1 Competitive Effects

In this subsection, I simulate counter-factuals in which one of the apps is shut down. For each pair of apps, I simulate market outcomes for an anonymous market in the first week of 2017 with different sets of (γ_{12}, ρ).[31] In Table 14, columns (2) and (3) present counter-factuals for the baseline estimates and columns (4) and (5) present counter-factuals for the estimates in the last column of Table 6 where we assume $\rho = 0$.

Table 14: Counter-factuals of Baidu Map and Amap

	Observed	Baseline		Assume $\rho = 0$	
	Outcomes	No Baidu Map	No Amap	No Baidu Map	No Amap
	(1)	(2)	(3)	(4)	(5)
$s_{BaiduMap}$	0.1056	0	0.139	0	0.107
s_{Amap}	0.0832	0.103	0	0.084	0
$t_{BaiduMap}$	0.0224	0	0.0314	0	0.022
t_{Amap}	0.0421	0.0497	0	0.0426	0
t_3	15.559	15.559	15.56	15.559	15.56
Diversion Ratio	-	35.04%	21.28%	0.18%	0.13%
$(\hat{\gamma_{12}}, \hat{\rho})$	-	(-1.15,0.7711)		(-0.02,0)	

Note:
1, The observed outcomes in column (1) are from an anonymous market in the first week of 2017.
2, $s_{BaiduMap}$ (s_{Amap}) are the number of active users of Baidu Map (Amap) divided by the number of active users of android cellphones. $t_{BaiduMap}$ (t_{Amap}, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphones.
3, Diversion ratio is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

The two sets of simulated outcomes are vastly different. When $\gamma_{12} = -0.02$

[31]Specifically, I invert out $\boldsymbol{\delta}$ for the two pairs of (γ_{12}, ρ) and then set δ^{μ}_{2mw} (δ^{μ}_{1mw}), the mean marginal utilities of app 2 (app 1), to be a very small number, -20, and simulate the market outcomes.

and $\rho = 0$, shutting down one app has almost no effect on the other. By contrast, when $\gamma_{12} = -1.15$ and $\rho = 0.7711$, the market share of Baidu Map would increase by 3.34 percentage points if I shut down Amap, and the market share of Amap would increase by 2 percentage points if I shut down Baidu Map. Consider the case of shutting down Amap when $(\gamma_{12}, \rho) = (-1.15, 0.7711)$. The inversion process reveals that there are 1.4% users that use both Baidu Map and Amap. Therefore, there are 6.92% users using Amap but not Baidu Map. When Amap is shut down, 3.34% out of the 6.92% unique users turn to Baidu Map. This implies an diversion ratio (in terms of unique active user) of $\frac{3.34}{6.92} = 48.3\%$. When we focus on time spent, diversion ratio is simply the increase in the total time spent on Baidu Map divided by the total time spent on Amap before its exit: $\frac{\Delta t_1}{t_2} = \frac{0.0314-0.0224}{0.0421} = 21.28\%$. In other words, when Amap exits the market, 21% of its time goes to Baidu Map and 79% of its time goes to offline activities and the generic app.[32] Note that the effects of shutting down Amap on Baidu Map are larger than the reverse. When Baidu Map is not available, the market share of Amap would only increase by 2 percentage points.

The counter-factual results for Baidu and Baidu Map with the baseline estimates are in Table 15. Shutting down Baidu Map has negligible effects on Baidu. However, shutting down Baidu would reduce Baidu Map's market

[32] As Baidu Map and Amap are close competitors, one may expect the diversion ratio to be higher. There are two justifications for this small diversion ratio. First, the diversion ratio in terms of unique active user is indeed much higher (48.3%). This discrepancy can be attributed to Amap's smaller satiation parameter relative to Baidu Map's, implying that a user who spends 1 hour on Amap would spend only 30 minutes on Baidu Map if they were to switch. Second, there are other map apps consumers can use such as Tencent Map, which is included in the generic app (app 3).

share by 0.86 percentage point and time spent on Baidu Map by more than 10%. It is therefore no surprise that Baidu, Inc. prioritizes the Baidu app as its core business.[33] A caveat to my findings is that the discovery process of apps is not modeled in this paper. Cross-promotion between apps developed by the same company is a widely used marketing strategy.[34] Promoting Baidu Map with Baidu will lead to a persistent large number of overlapping users if there are significant switching costs. Diversion ratio is -10% when Baidu Map exits the market. Even if Baidu Map is unprofitable, Baidu Inc. may still choose to maintain it due to its positive impact on Baidu app usage. This strategic rationale helps explain why tech conglomerates often develop and operate unprofitable apps for an extended time.

The counter-factuals for WeChat and Kwai with the baseline estimates are in Table 16. As expected, the competitive effects of WeChat on Kwai are larger than the reverse. Given that WeChat is the dominant player and Kwai is the entrant, it would be more interesting to focus on the effect of Kwai on WeChat. The market share of WeChat does not change in response to the exit of Kwai. This is because almost all Kwai users (11.7% out of 12.2%) already use WeChat. A diversion ratio of 29% means that about 29% of the time spent on Kwai comes from WeChat. The remaining 71% mostly comes from offline activities.[35] While the absolute value of 0.06 hours (3.8 - 3.74 hours)

[33]In the annual reports of Baidu, Inc., Baidu app, Baidu Map and other apps named after Baidu are in the "Baidu Core" business group.

[34]See The Ultimate Mobile Marketing Playbook by App Annie at https://www.appannie.com/en/insights/aso-app-store-optimization/ultimate-mobile-marketing-playbook/

[35]I assume away complementarity and correlated preference between app 1 (app 2) and the generic app.

Table 15: Counter-factuals of Baidu and Baidu Map

	Observed Outcomes (1)	No Baidu (2)	No Baidu Map (3)
s_{Baidu}	0.2459	0	0.259
$s_{BaiduMap}$	0.1056	0.097	0
t_{Baidu}	0.3222	0	0.3201
$t_{BaiduMap}$	0.0225	0.0201	0
t_3	15.279	15.284	15.28
Diversion Ratio	-	-0.74%	-10%
$(\hat{\gamma}_{12}, \hat{\rho})$		(0.1467,-0.0448)	

Note:
1, The observed outcomes in column (1) are from an anonymous market in the first week of 2017.
2, s_{Baidu} ($s_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t_{Baidu} ($t_{BaiduMap}$, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphones.
3, Diversion ratio is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

might appear modest for a typical Android smartphone user, it translates to an average increase of approximately 30 minutes ($\frac{0.06}{12.2\%} \approx 0.5$ hours) in daily WeChat usage for the 122 Kwai users among the 1000 Android smartphone users. For WeChat, competition from Kwai is significant. A caveat is that the competitive effects of Kwai on WeChat I estimated with data from 2017 is a lower bound. Usage of WeChat and that of Kwai have grown significantly since then. Tencent incorporated short-video and live-streaming features within WeChat to directly compete with Kwai and Tik Tok in the first quarter of 2020. The competitive effects will be much larger now.

Table 16: Counter-factuals of WeChat and Kwai

	Observed Outcomes (1)	No WeChat (2)	No Kwai (3)
s_{WeChat}	0.7725	0	0.779
s_{Kwai}	0.1221	0.222	0
t_{WeChat}	3.7391	0	3.7984
t_{Kwai}	0.2025	0.4257	0
t_3	11.6821	11.7173	11.6835
Diversion Ratio	-	5.98%	29.08%
$(\hat{\gamma}_{12}, \hat{\rho})$	-	(-0.08,0.42)	

Note:
1, The observed outcomes in column (1) are from an anonymous market in the first week of 2017.
2, s_{WeChat} (s_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t_{WeChat} (t_{Kwai}, t_3) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphones.
3, Diversion ratio is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

7.2 Decomposition: the Structural Model and the Index

Table 17 details the decomposition of one app's competitive effects on another, as defined in section 3.1 and compares the decomposition results with the descriptive index proposed in section 3.4. We would have more confidence in the index if the index and the decomposition results calculated from the full model are close.

As shown in Table 17, the index produces estimates reasonably close to the results from the full structural model despite the simplifying assumptions. One exception is the budget competition effect of Kwai on WeChat. The index (4.5) is 9 time the decomposition from the model (0.5). However, the index is quite accurate (4.61 versus 3.32) when we consider the budget competition effect of

Table 17: Functional Competition and Budget Competition

	Baidu Map and Amap	Baidu and Baidu Map	WeChat and Kwai
The Exit of App 1			
Budget Competition	0.0027	0.0126	3.3248
Functional Competition	7.875	-2.4075	220.1286
Total Effects on App 2	7.8777	-2.3949	223.4533
The Descriptive Index	0.0056	0.0432	4.6095
The Exit of App 2			
Budget Competition	0.0091	0.0191	0.5007
Functional Competition	8.8863	-2.2601	58.2947
Total Effects on App 1	8.8954	-2.2409	58.7954
The Descriptive Index	0.0056	0.0432	4.5124
t_1^*	0.0224	0.3222	3.7391
t_2^*	0.0421	0.0225	0.2025
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-1.15,0.7711)	(0.1467,-0.0448)	(-0.08,0.42)

Notes:
1, This table is based on data from an anonymous market in the first week of 2017.
2, The cells corresponding to the descriptive index is calculated according to (12) and then times 1000 so that they are comparable to results from the full model.

WeChat on Kwai. The key reason is that the market share of WeChat is more than 6 times that of Kwai, which violates the assumption that $\mu_1 = \mu_2 = 1$.

This is less of a concern now because of the spectacular growth of WeChat and Kwai since 2017. $\mu_1 = \mu_2 = 1$ is a more realistic assumption now. Based on recent data[36], a reasonable estimate for a representative smartphone user in China is that her average weekly time spent on WeChat in 2023 is approximately $t_1^* = 10$ hours, which is $\frac{10}{3.7391} = 2.67$ times the corresponding value in 2017. Her average weekly time spent on Kwai in 2023 is about $t_2^* = 5$ hours, which is $\frac{5}{0.2025} = 24.69$ times the corresponding value in 2017. If Kwai were to exit the market, the budget competition effect on WeChat is

[36]See https://ir.kuaishou.com/static-files/c7f60b19-c078-45e3-aca3-e7bae190a7d3 , https://lmtw.com/mzw/content/detail/id/227200 and https://www.questmobile.com.cn/research/report/1686624886410285058.

$1000 \times 5 \times \frac{10}{168-5} = 306.7$ hours for 1000 smartphone users. Note that this number is 0.5 hours (or 4.51 hours if we use the index) for 1000 smartphone users in Table 17. This significant discrepancy underscores the quadratic nature of budget competition.

Rather than taking the index at face value, we can use this index to gauge the order of magnitude of budget competition, which is often exaggerated or downplayed in business and legal settings. In the high-profile antitrust lawsuit filed by Qihu against Tencent in 2012, Tencent was accused of abusing its market dominance in the instant messaging market, where its software QQ had a market share of 80%–95% according to different measures. Tencent countered that the relevant market should encompass virtually all Internet companies and their software because they were all competing for user time. Tencent exaggerated budget competition to obfuscate its market dominance in the instant messaging market. I do not have reliable usage data of QQ or other major software products in 2013. However, it is highly unlikely that their usage would exceed the usage of apps analyzed in this paper. Hence, budget competition should be negligible in this lawsuit. Indeed, the court did not accept Tencent's market definition and stick to the market definition based on functions.

7.3 Extensions of the Index

In this section, I discuss two examples to show how this index can be useful. The examples show that the index can be combined with institutional knowledge ($\frac{\gamma_{12}}{\gamma_1}$, binge-watching, multi-stage budgeting) to more accurately gauge budget competition.

The decomposition of the gross diversion ratio in (9) suggests a way to combine the index with our belief of complementarity. Let use revisit the WeChat and Kwai example. For the sake of argument, assume $\frac{\gamma_{12}}{\gamma_1} = -0.05$ for WeChat and Kwai and we approximate $\frac{1}{\gamma_2}\frac{1}{\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3}}$ with $\frac{10}{168-5} = 0.061$. The gross diversion ratio is $-0.05 + 1.05 \times 0.061 = 0.014$. Therefore, a pair of complementary apps can be gross substitutes. Another way to use (9) is to calculate the threshold of $\frac{\gamma_{12}}{\gamma_1}$ above which the two apps are gross substitutes. In this case, the threshold is $\frac{-0.061}{1-0.061} = -0.065$. In other words, the diversion ratio implied by complementarity must be larger than 6.5% for WeChat and Kwai to be gross complements. Budget competition implies that being too "large" *per se* is a source of antitrust concern. Regulators can use this index as a first step to screen mergers of apps.

In 2017, Netflix CEO Reed Hastings stated, "We're competing with sleep"(Hern, 2017). Using this index, we can put a ballpark figure on this claim. Assuming that Netflix users spend 0.5 hours per day on Netflix[37] and 7 hours sleeping. The budget competition effect of Netflix on sleep is $\frac{0.5\times 7}{24-0.5} \approx 0.15$, about 9 minutes per day. Netflix users binge-watch. On a binge-watching day, assume a user spend 4 hours on Netflix and 5 hours sleeping. The budget competition effect of Netflix on sleep is larger: $\frac{4\times 5}{24-4} = 1$.This is more reasonable than 9 minutes. we can further refine our estimation. Recall the quote from Reed Hastings: "Think about if you didn't watch Netflix last night: What did you do? There's such a broad range of things that you did to relax and

[37]See the estimates by eMarketer at https://www.insiderintelligence.com/chart/232130.

unwind, hang out, and connect–and we compete with all of that."(Raphael, 2017) Let's assume a user spends 12 hours on work and related activities, leaving only 12 hours for Netflix, sleep, or leisure activities. Budget competition is $\frac{4\times5}{12-4} = 2.5$. This estimation should align more closely with Reed Hastings' statement: "We're competing with sleep." This is essentially a multi-stage budgeting model and related to our discussion regrading the choice of T in section 2.2.

8 Conclusion

The rapid development of the mobile Internet industry and its profound influence on our society warrant further understanding of this industry. This paper informs the public debate on antitrust issues in the mobile Internet industry. In this paper, I develop a discrete-continuous model of consumer demand for apps that allows for complements as well as substitutes, and incorporates a binding time constraint. I estimate the model with a weekly panel of app usage in the first quarter of 2017 in China. I validate the model by applying it to three representative pairs of apps: each featured an important aspect of the competition landscape in this industry (*a priori* substitutes, *a priori* complements, and a pair with an ambiguous relationship). I then define and theoretically decompose budget competition. Budget competition can dominate functional competition and a merger of complementary apps can hurt consumers. I propose a simple descriptive index to gauge budget competition. The index reveals that budget competition grows with the correlation between usage of apps and increases **quadratically** with time spent on apps. The index produce results similar to the decomposition results from the estimated structural models.

The demand model in this paper incorporates four desirable features: discrete-continuous decisions, interaction between products, budget constraints, and estimation with instruments. This model can further incorporate other notable features in the mobile Internet industry (for example, advertisement and two-sidedness) or be adapted to study consumer demand for other goods and services. One shortcoming of the demand model is that it does not accommodate dynamics. Future work could consider modeling dynamics of apps.

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A Reduced Form Evidence of Identification

The identification of complemenatrity (γ_{12}) and correlated preference (ρ) is from two sources: the overlapping user data and updates of apps as IV. In this section, I provide reduced-form evidence of their usefulness. Consider the following simple regression equation:

$$ln(q_{jt}) = a + b^{ols}_{jj'} ln(q_{j't}) + \varepsilon_{1jt} \tag{26}$$

where q_{jt} is the number of active user of app j in week t in the whole nation. $b^{ols}_{jj'}$ summarizes the co-movement between j and j' and is increasing in both $\gamma_{jj'}$ and $\rho_{jj'}$. When we have overlapping user data and updates, we can use the following two regressions:

$$ln(q_{jt} - c_{jj't}) = a + b^{c}_{jj'} ln(q_{j't} - c_{jj't}) + \varepsilon_{2jt} \tag{27}$$

$$ln(q_{jt}) = a + b^{iv}_{jj'} ln(q_{j't}) + \varepsilon_{3jt} \tag{28}$$

where $c_{jj't}$ is the number of overlapping user between j and j'. In equation (28), I use the update history of the iOS version of j' as instruments for $q_{j't}$. Specifically, I use the cumulative numbers of small, medium, and major updates of j'. Therefore, $b^{diff}_{jj'} = b^{ols}_{jj'} - b^{c}_{jj'}$ is the information we can get from the overlapping user data and $b^{bias}_{jj'} = b^{ols}_{jj'} - b^{iv}_{jj'}$ is the information we can get from the instruments. I then regress estimates of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'}, b^{diff}_{jj'}, b^{bias}_{jj'})$ on a category dummy which equals one if j and j' are in the same category defined by iResearch and zero otherwise. The categorization is based on functions and conforms to traditional definitions of a market (map, browser, music, etc.). Despite the criticism of categorizations in the introduction, they are still informative. A pair of apps in the same category should have a negative γ and a large ρ. If the estimates of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'}, b^{diff}_{jj'}, b^{bias}_{jj'})$ is correlated with this category dummy meaningfully, then we may conclude that the overlapping user data and the IV are useful.

I have update history of 84 apps and I run regressions on $83 \times 84 = 6972$ pairs of apps. Note that I have only 13 observations for each pair of apps because both overlapping user and update history are observed at the national level. I simulate 1000 samples of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'})$ for all pairs using the mean and variances from estimated equations (26), (27), and (28). Then I regress 1000 such samples of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'}, b^{diff}_{jj'}, b^{bias}_{jj'})$ on the category dummy. The mean and the 95% confidence interval of the coefficients from the 1000 regressions are in Table 18.

The coefficient in column (1) of Table 18 is significant and positive. This is because preferences for apps in the same category are often highly correlated. Instruments should remove at least some of the bias because of correlated preferences (ρ). This is what we see in column (2): a smaller and insignificant coefficient. A cautionary note is that the coefficient in column (4) is not significant. One might be concerned about the weak IV problem given that the confidence interval in column (2) is much larger than that in column (1). The results in Table 19 are conditional on F statistic larger than 10. There are 50 apps with a F statistics larger than 10 and hence $50 \times 84 - 50 = 4150$ observations. The confidence intervals are smaller in column (2) in Table 19 and the coefficient is still insignificant. Therefore the insignificance of the coefficient in column (2) in Table 18 is not driven by weak instruments. The relationship between $b^c_{jj'}$ and the structural parameters $\gamma_{jj'}$ and $\rho_{jj'}$ is complicated. The co-movement of *the exclusive users* for apps in the same category is much larger than the co-movement of their *total active users*. One explanation is that the growth of competing apps mostly comes from exclusive users rather than overlapping users. In other words, users who did not use A nor B started using A or B but not both. Overall, the reduced form results indicate that the overlapping user data and updates are useful for our identification.

Table 18: Reduced Form Evidence of Identification

	$b^{ols}_{jj'}$ (1)	$b^{iv}_{jj'}$ (2)	$b^{c}_{jj'}$ (3)	$b^{bias}_{jj'}$ (4)	$b^{diff}_{jj'}$ (5)
Same-Category	0.1636	0.0246	0.3784	0.139	-0.2148
	[0.087, 0.239]	[-0.269, 0.337]	[0.298, 0.456]	[-0.183, 0.45]	[-0.325, -0.103]
N	6972	6972	6972	6972	6972
R^2	0.0004	0.0000	0.0012	0.0001	0.0005

Note: The coefficients and the 95% confidence interval are based on 1000 simulations.
Data Source: iResearch and the author's calculations.

B Covariates in μ_1 and μ_2

The covaraites in μ_1 and μ_2 are market fixed effects and week fixed effects. In Table 20, I provide covariates from the main specification of Baidu Map and Amap (Column (3) in Table 6). In the following table, I report week fixed effects and aggregate market fixed effects to gender and age groups. The

Table 19: Identification with Strong Instruments

	$b^{ols}_{jj'}$ (1)	$b^{iv}_{jj'}$ (2)	$b^{bias}_{jj'}$ (3)
Same-Category	0.1163	0.0375	0.0788
	[0.044, 0.186]	[-0.056, 0.129]	[-0.041, 0.197]
N	4150	4150	4150
R^2	0.0004	0.0000	0.0001

Note: The coefficients and the 95% confidence interval are based on 1000 simulations. The regressions are based on the group of apps with a first stage F statistic larger than 10. Data Source: iResearch and the author's calculations.

results are reasonable: users between 31 and 35 and male users derive higher utility from map apps because they are more likely to own and drive a car in China.

C Budget Competition

The intermediate bundle (t^i_0, t^i_1, t^i_3) defined by (8) is easy to calculate. The functional competition in Table 2 is $t^i_1 - t^o_1$. After this step, we can calculate how much time is left to be allocated as $\Delta T = T - t^i_0 - t^i_1 - t^i_3$. The intermediate bundle can be seen as the result of utility maximization over t_0, t_1, and t_3 subject to a time budget of $t^i_0 + t^i_1 + t^i_3$. Note that apps 0, 1, and 3 are independent. We can solve for the final bundle as the same utility maximization problem subject to a time budget of $t^i_0 + t^i_1 + t^i_3 + \Delta T$. The following two lemmas are useful when calculating the final bundle. The budget competition effect of app 2 on app 1 is $t^f_1 - t^i_1$.

Lemma 1. *For J independent apps that are used, when there are extra time*

ΔT*, the increase in time spent on app* j *is* $\Delta t_j = \Delta T \frac{1}{\gamma_j} \frac{1}{\sum_{k=1}^{J} \frac{1}{\gamma_k}}$.

Proof. From the FOCs of the old bundle, we have

$$\mu_j + \gamma_j t_j^0 = \mu_k + \gamma_k t_k^0 \Rightarrow t_k^0 = \frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^0.$$

Similarily, we have $t_k^1 = \frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^1$. Because of the time constraint, we have

$$\sum_{l=1}^{J} t_l^0 = T \Rightarrow \sum_{k=1}^{J} (\frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^0) = T \Rightarrow t_j^0 = \frac{T - \sum_k \frac{\mu_j - \mu_k}{\gamma_k}}{\gamma_j (\sum_{k=1}^{J} \frac{1}{\gamma_k})}$$

The budget constraint with extra time ΔT is

$$\sum_{l=1}^{J} t_l^1 = T + \Delta T \Rightarrow \sum_{k=1}^{J} (\frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t_j^1) = T + \Delta T \Rightarrow t_j^1 = \frac{T + \Delta T - \sum \frac{\mu_j - \mu_k}{\gamma_k}}{\gamma_j (\sum_{k=1}^{J} \frac{1}{\gamma_k})}$$

Therefore we have

$$\Delta t_j = t_j^1 - t_j^0 = \Delta T \frac{1}{\gamma_j} \frac{1}{\sum_{k=1}^{J} \frac{1}{\gamma_k}}$$

□

Lemma 2. *When an app* q *is used because of the extra time* ΔT*,* $t_q^1 = \frac{T + \Delta T - \sum_k \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q (\sum_k \frac{1}{\gamma_k})} \leq \frac{\Delta T}{\gamma_q (\sum_k \frac{1}{\gamma_k})}$

Proof. Because q was not used ($t_q^0 = 0$), we have

$$\mu_q \leq \mu_k + \gamma_k t_k^0 \Rightarrow \frac{\mu_q - \mu_k}{\gamma_k} \geq t_k^0 \Rightarrow T \leq \sum_k \frac{\mu_q - \mu_k}{\gamma_k}$$

The FOCs of the new bundle are

$$\mu_q + \gamma_q t_q^1 = \mu_k + \gamma_k t_k^1 \Rightarrow t_k^1 = \frac{\mu_q - \mu_k}{\gamma_k} + \frac{\gamma_q}{\gamma_k} t_q^1$$

Combined with the new time constraint, we have

$$T + \Delta T = \sum_k \frac{\mu_q - \mu_k}{\gamma_k} + \frac{\gamma_q}{\gamma_k} t_q^1 \Rightarrow t_q^1 = \frac{T + \Delta T - \sum_{k \neq q} \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q (\sum_k \frac{1}{\gamma_k})}$$

Because $T \leq \sum_k \frac{\mu_j - \mu_k}{\gamma_k}$, we have

$$\frac{T + \Delta T - \sum_k \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q (\sum_k \frac{1}{\gamma_k})} \leq \frac{\Delta T}{\gamma_q (\sum_k \frac{1}{\gamma_k})}.$$

□

D Derivation of the Descriptive Index

The FOCs of this user at the observed usage level are

$$1 - 0.001 t_0^* = 1 - 0.001(T - t_1^* - t_2^*) = 1 + \gamma_1 t_1^* = 1 + \gamma_2 t_2^* \tag{29}$$

We have

$$t_1^* = \frac{0.001(t_2^* - T)}{\gamma_1 - 0.001} = \frac{\gamma_2}{\gamma_1} t_2^*$$

When app 2 exits the market, the new FOCs are

$$1 - 0.001t_0' = 1 - 0.001(T - t_1') = 1 + \gamma_1 t_1'$$

We have

$$t_1' = \frac{-0.001T}{\gamma_1 - 0.001}$$

$$\begin{aligned}
t_1' = \frac{-0.001T}{\gamma_1 - 0.001} &= \frac{-0.001T + 0.001t_2^*}{\gamma_1 - 0.001} - \frac{0.001t_2^*}{\gamma_1 - 0.001} \\
&= t_1^* - \frac{0.001t_2^*}{\gamma_1 - 0.001} \\
&= t_1^* + t_2^*(\frac{-0.001}{\gamma_1 - 0.001}) \\
&= t_1^* + t_2^*(\frac{t_1^*}{t_0^* + t_1^*})
\end{aligned}$$

where the last equality is from $-0.001t_0^* = \gamma_1 t_1^* = \gamma_2 t_2^*$.

Table 20: Covariates of Baidu Map and Amap in Taste Parameters

Covariates	Baidu Map	Standard Error	Amap	Standard Error
Week (02)	-0.0078	0.0002	-0.0452	0.0002
Week (03)	0.0275	0.0002	0.0026	0.0001
Week (04)	-0.0349	0.0000	-0.0466	0.0001
Week (05)	0.0276	0.0001	-0.0466	0.0002
Week (06)	0.0649	0.0002	-0.0466	0.0002
Week (07)	0.0631	0.0001	0.0053	0.0001
Week (08)	0.0657	0.0001	-0.0016	0.0002
Week (09)	0.0465	0.0001	-0.0138	0.0001
Week (10)	-0.0838	0.0002	-0.0138	0.0002
Week (11)	0.0944	0.0003	-0.0138	0.0004
Week (12)	0.107	0.0002	-0.0066	0.0003
Week (13)	0.1295	0.0004	-0.0138	0.0003
Female	0.265	-	0.4661	-
Male	0.3792	-	0.543	-
Age (<=24)	0.2557	-	0.5019	-
Age (25~30)	0.4124	-	0.6479	-
Age (31~35)	0.552	-	0.7084	-
Age (36~40)	0.2724	-	0.4142	-
Age (>=40)	0.1185	-	0.2504	-

Notes:
1, This table provides parameters of covariates in μ_1 and μ_2 corresponding to the column (3) of Table 6.
2, The coefficients of gender and age groups are the simple average of market fixed effects with corresponding characteristics.
Data Source: The author's calculations.