Competing for Time: A Study of Mobile Applications*

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Abstract

Apps compete for limited user time regardless of their functions. I develop a discrete-continuous demand model with a binding time constraint. I estimate the model on three pairs of apps (including substitutes and complements) with joint app usage in China. I use updates to disentangle complementarity from correlated preferences. I decompose competition into functional competition and budget competition, the latter of which captures the effects of a binding time constraint. Budget competition can dominate functional competition and a merger of complements can hurt consumers. A model-free metric is proposed to gauge budget competition.

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Instagram can hurt us.

–Mark Zuckerberg, Facebook CEO, 2012

We're competing with sleep, on the margin.

–Reed Hastings, Netflix CEO, 2017

1 Introduction

Antitrust authorities have been struggling to deal with mergers of free apps (Wu, 2017).[1] None of the major acquisitions by tech giants were blocked in recent years in the United States (Cabral, 2021).[2] Scholars believe that the reason is that antitrust authorities lack the tools to analyze mergers of apps.[3] The UK Office of Fair Trading (OFT) approved the acquisition of Instagram by Facebook in 2012 partly on the ground that, in the market for camera apps, Facebook would still face competition from other photo apps after the merger.[4] The market definition that Instagram is a photo app and Facebook is not is certainly debatable. Zuckerberg would disagree. When contemplating the acquisition, Zuckerberg wrote to his CFO to explain the motivation:"If they grow to a large scale the could be very disruptive to us". Zuckerberg saw one social networking app (Instagram) challenging another (Facebook) and that acquisition can "neutralize a potential competitor"[5]. In another email sent days before the announcement of the merger, he wrote: "Instagram can hurt us." From a different position, Tim Wu, the famous legal scholar, and Chris Hughes, a co-founder of Facebook, also believe Instagram and Facebook are competitors and antitrust authorities should reverse this and other similar decisions (Wu, 2018; Hughes, 2019). There are arguments for both sides. Considering that users can share Instagram photos to Facebook, the

[1]See Scott Morton *et al.* (2019), Furman *et al.* (2019), European Commission. Directorate General for Competition. (2019), and Cabral (2021) for related discussions.

[2]The same is true for China until a recent wave of antitrust enforcement. In July 2021, a proposed merger of Douyu and Huya, two live-streaming apps, was blocked because of their vertical relationship with Tencent.

[3]In his opinion piece in the Washington Post, Tim Wu (Wu, 2018) argues:

> "Our standards for assessing mergers, fixated on consumer prices, were a poor match for the tech economy and are effectively obsolete."

In the report commissioned by the Stigler Committee on Digital Platforms, Scott Morton *et al.* (2019) proposes

> "The law needs better analytical tools to take into account the impact of potential and nascent competitors and competition. Market definition will vary according to what consumers are substituting between[......]."

[4]See Anticipated acquisition by Facebook Inc of Instagram Inc, ME/5525/12 (Office of Fair Trading, 22 August 2012) at https://assets.publishing.service.gov.uk/media/555de2e5ed915d7ae200003b/facebook.pdf.

[5]Those emails were revealed during the House antitrust subcommittee's hearing on antitrust issues in July 2020. See the story "'Instagram can hurt us': Mark Zuckerberg emails outline plan to neutralize competitors" at https://www.theverge.com/2020/7/29/21345723/facebook-instagram-documents-emails-mark-zuckerberg-kevin-systrom-hearing

two apps are also complements.[6] Different assumptions about the competitive relationship between Facebook and Instagram (complements, substitutes, or independents) can lead to different antitrust decisions. What we need is a quantitative method that can estimate complementarity/substitutability between apps from data.

Competition among apps is further complicated by the binding time constraint. When asked about competition at an Netflix's earnings call in 2017, Reed Hastings replied with the opening quote (Hern, 2017). He explained it in another occasion (Raphael, 2017): "Think about if you didn't watch Netflix last night: What did you do? There's such a broad range of things that you did to relax and unwind, hang out, and connect–and we compete with all of that." Hastings highlighted the role of time constraint in competition analysis. In this paper, I refer to it as "budget competition" to distinguish it from "functional competition". Users have at most 24 hours per day. A minute spent on Tik Tok is a minute not spent on WeChat. Budget competition has been invoked in the landmark antitrust case *Qihu v. Tencent* in 2013 to expand the relevant market.[7] Tencent argued that its instant-messaging software QQ competes with all other Internet companies for user attention (time).[8] This claim is trivially true for all apps/software and offline activities regardless of complementarity or substitutability. What matters is the significance of budget competition. When budget competition is large enough, a merger of complements can hurt consumers. To quantify budget competition, we must estimate a structural model of demand with a binding time constraint, in addition to allowing for complementarity/substitutability.

The difficulty in estimating competitive relationships among apps is twofold. First, modeling competition without price is a daunting task. There are no price variations, and therefore we cannot estimate price elasticities. Functional definitions do not work. WeChat, the flagship app of Tencent, is classified as "Social Networking" by the Apple App Store and "Communication" by the Google Play Store as of 2022. Users know that WeChat is more than the two definitions: it is also a mobile payment app, a publishing platform, a platform of mini programs, and so on. It is everything and it is competing with everyone. Adding a budget constraint would further complicate utility maximization problems and hence demand estimation. Second, estimating complementarity/substitutability is difficult. Taste for variety and correlated preferences can confound the estimation of complementarity. We need more than aggregate market share data to separate complements from substitutes.

In this paper, I propose a model of time allocation to apps. The model features a quadratic utility function to capture the discrete-continuous nature of app usage and allow

[6] In the public announcement of the acquisition, Zuckerberg said:"We believe these are different experiences that complement each other." See the public announcement at https://investor.fb.com/investor-news/press-release-details/2012/Facebook-to-Acquire-Instagram/default.aspx

[7] For a brief introduction of this case, see https://www.pymnts.com/cpi_posts/qihoo-360-v-tencent-first-antitrust-decision-by-the-supreme-court/ and https://enipc.court.gov.cn/en-us/news/view-22.html.

[8] Hence the relevant market in this case should include all major Internet companies and their software. The Supreme People's Court rejected this argument with qualitative analysis in the final adjudication in 2013.

for substitutes as well as complements (Thomassen *et al.*, 2017)[9]. I add a binding time constraint to study budget competition. In this model, an app is described by a taste parameter, a satiation parameter, and interaction parameters with other apps. The taste parameter is the marginal utility at zero usage, and the satiation parameter determines how fast the marginal utility depreciates as an user spends more time on the app. The interaction parameters are cross partial derivatives that describe the functional interdependence/overlapping between apps: if the interaction parameter between a pair of apps is positive, then they have interdependent functions and are complements; otherwise, they have overlapping functions and are substitutes. Taste parameters have random components that can be correlated across apps. Users allocate their time to apps and offline activities subject to a time constraint. Quadratic utility functions are second-order approximations to any utility functions. Therefore, my model nests the random coefficients discrete choice model of Berry *et al.* (1995) as a special case with only taste parameters.

I formally define budget competition and functional competition. Functional competition captures the fact that if two apps offer similar functions, using one would reduce the (marginal) utility of the other. Budget competition captures the fact that all apps are competing for the limited time of users. I then decompose gross diversion ratio into two parts: diversion because of functional competition and diversion because of budget competition. An implication of this decomposition is that when budget competition dominates functional competition, a pair of functionally complementary apps can be gross substitutes. The likelihood of this scenario depend on a nonlinear function of structural parameters. I propose a model-free metric to gauge budget competition of app 2 on app 1, $\frac{t_2 t_1}{T-t_2}$ where t_1 and t_2 are the time spent on the two apps and T is the total time budget. Budget competition is increasing in $t_1 t_2$ and, by extension, the correlation between t_1 and t_2. With this metric, we can put a ballpark figure on budget competition with aggregate usage data or a simple survey of users.

I estimate the model using weekly market-level app usage data from China in the first quarter of 2017. This is the first academic paper to use this type of data, to the best of my knowledge.[10] Markets in this data set are demographic groups in China defined by age, gender, and province. I observe the number of active users and usage (time spent) of popular apps and Android smartphone. The active user data help me identify taste parameters of apps and the usage data help me identify satiation parameters. In addition, for each pair of apps, I observe the number of common users who use both apps in a week. The common user data are informative, but these data are not sufficient for the identification of complementarity/substitutability.

The econometric challenge is to separate correlated preferences from complementar-

[9]Lewbel & Nesheim (2019) also use a quadratic utility model.

[10]A concurrent paper by Kawaguchi *et al.* (2022) uses national usage data of apps from Japan. One key difference is that they do not have common user data, which is crucial to my identification.

ity/substitutability. The observation of common users in two apps can be the result of complementarity between the two apps, or the fact that the preferences of the two apps are positively correlated due to unobserved characteristics. My identification strategy is based on an "old definition" of complements (substitutes)[11]: if users spend more time on an app due to an exogenous increase in its utility, the (marginal) utility of its complements (substitutes) would increase (decrease). Updates of an app should affect the utility of this app but not the *utilities* of other apps. However, updates of an app could change the *usage* of other apps through complementarity/substitutability. This is similar to the strategy used in Gentzkow (2007). I use GMM estimation *a la* Berry *et al.* (1995) such that I could utilize instrument variables (IVs).

My model can recover diverse competition relationships. I apply this model to three representative pairs of apps: a pair of substitutes *a priori* (Baidu Map and Amap), a pair of complements *a priori* (Baidu and Baidu Map), and a pair of apps with an ambiguous relationship (WeChat and Kwai)[12]. In each case, the estimated results are reasonable. WeChat and Kwai were weak substitutes though they did not offer similar functions at the time. This can be interpreted as one social media app competing with another social media app, as with Instagram and Facebook. IVs are crucial to my estimation. When I assume away correlated preferences and rely only on the common user data for identification, Baidu Map and Amap are estimated to be almost independent apps.

I simulate counter-factuals to see what happens to the other app if one of the apps is shut down. Consulting firms publish reports analyzing market share of apps based on time spent rather than active users.[13] In Figure 1, I plot the market shares of tech giants in terms of time spent on their apps. It is only natural for Tencent to ask if ByteDance or other firms/apps competed time away from its apps. My simulations can answer this question. I find that Kwai users would increase their time spent on WeChat on average by 30 minutes if Kwai is shut down. In other words, Kwai competes 30 minutes away from WeChat each week. The competition pressure is significant considering that Kwai was a young app and they did not offer similar functions at the time. I calculate diversion ratios in terms of time spent. In the WeChat and Kwai example, if Kwai is shut down, 29% of its time will be diverted to WeChat; if WeChat is shutdown, 6% of its time will be diverted to Kwai.

I then calculate functional competition effects and budget competition effects using the structural estimates. Budget-competition effects are negligible (less than 0.02 hour for 1000 users) for the first two pairs of apps (Baidu Map and Amap, Baidu and Baidu Map). The

[11]Samuelson (1974) discusses various definitions of complements and substitutes.

[12]WeChat and Kwai did not offer similar functions at the time. The relationship is similar to that between Instagram and Facebook before the merger.

[13]For example, Nielsen (https://www.nielsen.com/us/en/insights/article/2014/smartphones-so-many-apps-so-much-time/) and QuestMobile (https://www.questmobile.com.cn/research/report-new/118).

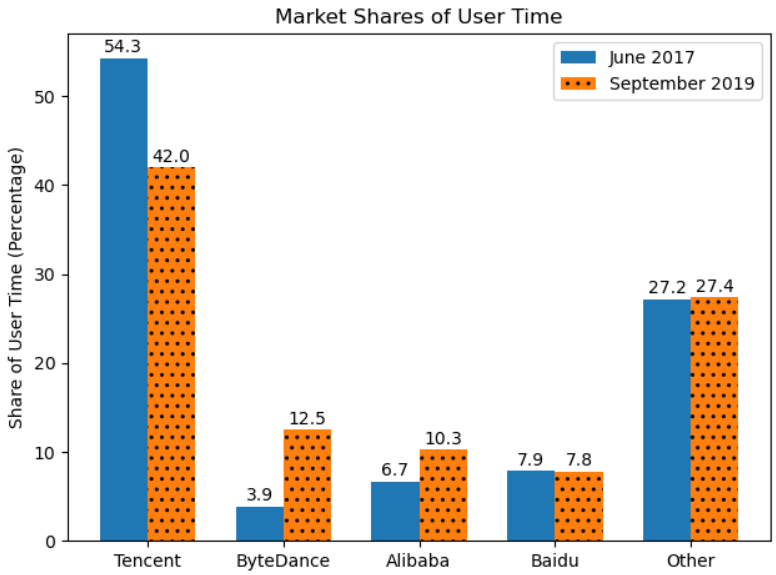


Figure 1: Market Shares of Tech Giants in China
Note: Market shares are calculated based on time spent on apps developed by each tech giants.
Data Source: Quest Mobile.

reason is that time spent on Baidu Map and Amap are small. This suggests that if the apps of interest are "small" (in terms of time spent), researchers can model the demand of these apps without a time constraint and still capture virtually all the competitive effects. The budget-competition effect is orders-of-magnitude larger (3.3 hour for 1000 users) for WeChat and Kwai because user preferences for the two apps are positively correlated, and a large number of users spend a substantial amount of time on both them. The budget-competition effect between WeChat and Kwai will be orders-of-magnitude larger because of the spectacular growth of WeChat and Kwai since 2017. I compare budget competition calculated from the estimated models and budget competition calculated using the model-free metric. The two sets of results are close in terms of orders of magnitude despite their different assumptions. I use several examples (WeChat and Kwai, Netflix and sleep, Facebook and Instagram) to show how this simple metric can provide insight into competition. This metric can be combined with institutional knowledge (complementarity, binge-watching, multi-stage budgeting, etc) to get more accurate estimates of budget competition. The decomposition also highlights that being too "large" *per se* is a source of antitrust concern when analyzing mergers of apps.

I combine the estimated demand model with a supply model to simulate mergers of apps. The demand and supply model highlights the fact that prices, ad load, and quality are isomorphic in both the utility function and the profit function and conceptually the same thing. With this observation, I propose a different version of the hypothetical monopolist test which considers a "Small but Significant and Non-transitory Increase in Comprehensive Price" (SSNICP), where the comprehensive price is the sum of the hourly price and the hourly ad revenue of an app. I back out ad revenue per hour from profit maximization conditions. Merger simulation shows that when competing apps (functional competition or budget competition) merge, prices (ad load) increase and consumer welfare decreases. Despite the fact that WeChat and Kwai had no similar functions in the first quarter of 2017, the comprehensive price of WeChat and the comprehensive price of Kwai would increase by 0.158 (1.7%) and 2.4 (61.7%) and the welfare of 1000 simulated smartphone users in one week could decrease by 947 yuan if the two apps merge. The results partly explain Tencent's 2 billion dollar investment in Kwai in late 2019.

This paper contributes to the emerging literature on mobile applications. Due to data limitations, researchers have mostly focused on the supply side of apps (Liu *et al.*, 2013; Yin *et al.*, 2014; Bresnahan *et al.*, 2014b,a; Liu, 2017; Wen & Zhu, 2017; Ershov, 2018; Leyden, 2019). The demand side for apps is either absent in the papers or described with aggregate ranking or downloads data from app stores (Carare, 2012; Ghose & Han, 2014; Li *et al.*, 2016; Yi *et al.*, 2017; Le Guel *et al.*, 2020; Deng *et al.*, 2020).[14] An immediate predecessor of this paper is Han *et al.* (2016). They adopt a multi-nominal discrete-continuous extreme value (MDCEV) model developed by Bhat (2005) and allow for correlation in utilities between different apps by adding a factor analytic structure. With individual level panel data from Nielsen KoreanClick, they estimate positive or negative correlations in preferences across apps. However, substitutes or complements are not modeled in their paper. As these authors have noted in their paper, the correlation of preferences between Naver and Daum and that between Kakao Talk and Kakao Story are estimated to be positive and large. However, common sense suggests that the first pair are substitutes and the second pair are complements. By contrast, my paper explicitly disentangles substitutability/complementarity from correlated preferences with the common user data and IVs. My model also differs from Han *et al.* (2016) in using market level data. Though these data are widely used in the industry, to the best of my knowledge, this is the first instance of using market-level data of app usage in academic research.

A concurrent paper by Kawaguchi *et al.* (2022) simulate mergers of apps. They estimate demand and supply for apps in two categories with usage and advertising data from Japan. The demand in their paper is more restrictive. The differences between their model and

[14] Both Wu *et al.* (2022) and Lee (2018) use a panel of individual usage of smartphone. However, both observe usage of categories rather than apps. Lee (2018) estimates the demand for smartphone. Wu *et al.* (2022) uses a hidden Markov model to analyze what motivates mobile app usage.

mine highlight the trade-off between flexible competition patterns and scalability. Their merger results are also different from mine. According to their results, a merger among the top 10 social media apps in Japan has negligible effects on surplus. Aridor (2022) studies addiction to social media apps with individual apps usage from an experiment. Aridor (2022) is complementary to my work because he focuses on the dynamic aspect of app usage that is not in my paper.

In terms of methodology, this paper is a second-order extension of Berry *et al.* (1995). My model is the first to combine four appealing features into a model of consumer demand: discrete-continuous decisions, interactions between products, budget constraint, and estimation with instruments. This paper relates to the literature on the demand of differentiated goods in economics and marketing especially when complementarity is of interest (Kim *et al.*, 2002; Nair *et al.*, 2005; Song & Chintagunta, 2006, 2007; Mehta, 2007; Gentzkow, 2007; Thomassen *et al.*, 2017; Ershov *et al.*, 2018; Vélez-Velásquez, 2019; Lewbel & Nesheim, 2019; Wang, 2020). Unlike Gentzkow (2007), this paper models not only the extensive margin (which products are chosen) but also the intensive margin (quantities of chosen products) of consumer decisions. This is especially important if we want to estimate complementarity. Consumers buy two boxes of cereal with different flavors because of taste for variety (decreasing marginal utility) rather than complementarity. A discrete choice model with bundles of different products cannot identify complementarity from taste for variety. Taste for variety is captured by satiation parameters and can be estimated with usage data in my paper. This paper also contributes to the study of time allocation in transportation research (Kitamura, 1984; Bhat, 2005; Pawlak *et al.*, 2015, 2017; Bhat, 2018) by directly estimating relationships between activities. Furthermore, this model is a flexible second-order approximation to consumer decisions and hence can be adapted to study other topics.

Finally, this paper contributes to the policy debate on regulating the digital economy (Furman *et al.*, 2019; European Commission. Directorate General for Competition., 2019; Scott Morton *et al.*, 2019). To the best of my understanding, this paper and Kawaguchi *et al.* (2020) are the first papers to simulate mergers of mobile applications. A key challenge in analyzing the digital economy is that the digital economy is characterized by free services and existing economic tools require prices. Complements and substitutes are defined with compensated cross-price elasticities and market power is defined with prices as well. Market power exists even though prices are zero. This paper extends economic tools to incorporate important features of apps, including zero (or negative) prices. In this paper, price and advertising are both (linear) components of quality. My analysis shows that market power exists and can be measured even when prices are zero. My decomposition of competition provides a new theory of harm to consumers: a merger of complementary apps can hurt users if budget competition dominates functional competition.

2 Model

2.1 The Baseline Model

A consumer $i = 1, 2, , ..., I$ allocates her time T to J apps and an outside option denoted by $j = 0$. The utility from an allocation described by $\mathbf{t} = [t_{i0}, t_{i1}, t_{i2}, ..., t_{iJ}]'$ where t_{ij} is the amount of time allocated to option $j = 0, 1, 2, ..., J$ is given by

$$U(\mathbf{t}) = \boldsymbol{\mu}'\mathbf{t} + 0.5\mathbf{t}'\boldsymbol{\Gamma}\mathbf{t} \tag{1}$$

where

$$\boldsymbol{\mu} = [\mu_{i0}, \mu_{i1}, ..., \mu_{iJ}]'$$

and

$$\boldsymbol{\Gamma} = \begin{bmatrix} \gamma_{i0} & \gamma_{i01} & \cdots & \gamma_{i0J} \\ & \gamma_{i1} & \cdots & \gamma_{i1J} \\ & & \ddots & \vdots \\ & & & \gamma_{iJ} \end{bmatrix}.$$

$\boldsymbol{\mu}$ is a $(J+1) \times 1$ vector of first order parameters and $\boldsymbol{\Gamma}$ is a $(J+1) \times (J+1)$ symmetric matrix of second order parameters. The marginal utility of app j is

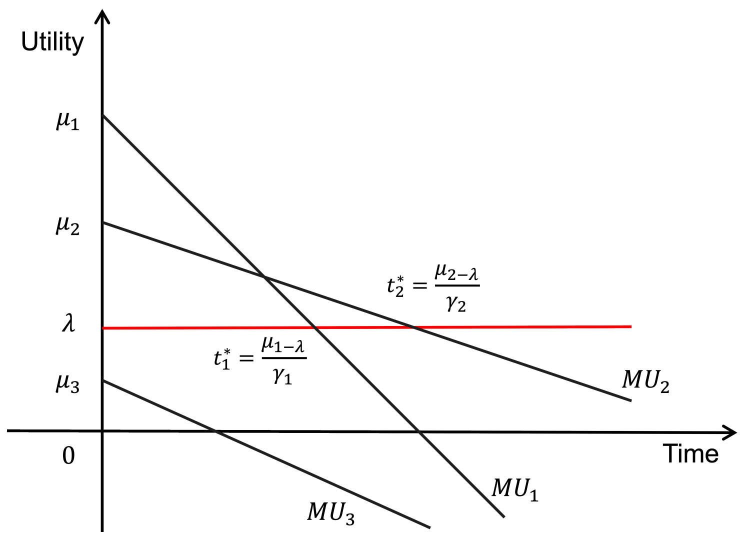
$$MU_{ij} = \mu_{ij} + \gamma_{ij} t_{ij} + \sum_{j' \neq j} \gamma_{ijj'} t_{ij'}.$$

The marginal utility of app j consists of three components. The first term μ_{ij} is the marginal utility of app j at zero usage and it will be referred to as the taste parameter of app j. γ_{ij} in the second term determines how MU_{ij} changes as a user spends more time on app j. Therefore, γ_{ij} should be negative and will be referred to as the satiation parameter of app j. The last term captures the impact of app j' on app j: if parameter $\gamma_{ijj'} > 0$, then MU_{ij} is increasing in $t_{j'}$ and they are complements; otherwise, they are substitutes.[15] Therefore, the interaction parameter $\gamma_{ijj'}$ determines if j and j' are likely to be used together.

At the optimal level $\mathbf{t}^*$, the marginal utilities of chosen apps should be equalized. Denote this as λ. Zero usage arises naturally when the marginal utility at zero is too small, i.e., $\mu_{ij} < \lambda$. In Figure 2, I plot three apps with different combinations of μ_j and γ_j without considering $\gamma_{jj'}$ for now. Intuitively, μ_j determines if an app is used and conditional on being used, γ_j determines the time spent on app j.

I choose the quadratic utility function because it naturally models the discrete-continuous

[15] The modern definition is based on compensated cross-price elasticities. Samuelson (1974) discusses various definitions of complements and substitutes.



Note: This graph plots the marginal utility of three apps: app 1, app 2, and app 3. λ is the marginal utility at the optimal allocation. For simplicity, I ignore $\gamma_{jj'}$. MU is the marginal utility of each app.

Figure 2: Marginal Utilities and Optimal Allocation of Time

nature of app usage and the complementarity/substitutability between apps. Despite the advantages of the quadratic utility function, the size of $\mathbf{\Gamma}$ increases quadratically in J. Therefore, instead of analyzing 100 apps in one model, which involves a gigantic matrix $\mathbf{\Gamma}$, I analyze a smaller model with more assumptions and only two apps of interest. Two is certainly not an ideal number. However, many mergers in the mobile Internet industry are about two apps (for example, Facebook's acquisition of Instagram). The only limit on how many apps researchers can analyze is computational resources and "thickness" of data (common user data for all possible pairs of apps).[16]

2.2 A Simplified Model

In the model to be estimated, there are four options $j = 0, 1, 2, 3$, where $j = 1, 2$ are the two apps of interest and $j = 0$ is the option of not using a smartphone and $j = 3$ is a generic app which is to use any other app. The utility maximization problem of consumer i in market

[16] One may also be concerned about the number of exclusion restrictions. This is not an issue if we use updates as IV. The update history of each app will be interacted with residuals of all other apps and *vice versa*. Therefore, the number of exclusion restrictions increases quadratically as well.

$m = 1, 2, ..., M$ is

$$\max_{t_{i0m}, t_{i1m}, t_{i2m}, t_{i3m} \geq 0} t_{i0m} - 0.0005 t_{i0m}^2 + \sum_{j=1}^{2} \mu_{ijm} t_{ijm} + 2 t_{i3m} + \frac{1}{2} \sum_{j=1}^{3} \gamma_{ijm} t_{ijm}^2 + \gamma_{12} t_{i1m} t_{i2m} \tag{2}$$

$$s.t. \quad t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = 168$$

I add more assumptions compared to (1). To normalize the level of the utility function, I assume $\mu_{i0m} = 1$. I assume $\mu_{i3m} = 2 > \mu_{i0m}$ because the market shares of $j = 3$ are always 1. Because the time spent on $j = 0$ is a residual term ($t_0 = 168 - t_1 - t_2 - t_3$) in the model, I assume γ_{i0m} be a non-positive constant -0.001 ($\frac{1}{2} \times 0.001 = 0.0005$). I also assume $\gamma_{10} = \gamma_{20} = \gamma_{13} = \gamma_{23} = 0$ because those who use either app 1 or app 2 will always use the generic app and spend some time on offline activities.

168 is the total number of hours in a week and the time scope of this utility function. This is a choice imposed by the data structure: I happen to observe weekly usage. One can certainly consider utility functions defined over a month, a day, an hour, or a second if data permits. The estimated demand models will be different but valid under their respective time scope. For example, when we model usage for each second, all apps are substitutes.[17] We can often observe more dis-aggregate usage with survey data.

2.3 Consumer Heterogeneity

Consumers have different preferences for apps. μ_{ijm} and γ_{ijm} are parameterized as

$$\mu_{i1m} = \mathbf{x}_m \boldsymbol{\beta}_1^\mu + \xi_{1m}^\mu + \varepsilon_{i1m} = \delta_{1m}^\mu + \varepsilon_{i1m} \tag{3}$$

$$\mu_{i2m} = \mathbf{x}_m \boldsymbol{\beta}_2^\mu + \xi_{2m}^\mu + \varepsilon_{i2m} = \delta_{2m}^\mu + \varepsilon_{i2m} \tag{4}$$

$$\gamma_{i1m} = \mathbf{x}_m \boldsymbol{\beta}_1^\gamma + \xi_{1m}^\gamma = \delta_{1m}^\gamma \tag{5}$$

$$\gamma_{i2m} = \mathbf{x}_m \boldsymbol{\beta}_2^\gamma + \xi_{2m}^\gamma = \delta_{2m}^\gamma \tag{6}$$

$$\gamma_{i3m} = \mathbf{x}_m \boldsymbol{\beta}_3^\gamma + \xi_{3m}^\gamma = \delta_{3m}^\gamma \tag{7}$$

where $\mathbf{x}_m$ is a set of exogenous market level variables. I follow Berry *et al.* (1995) and Nevo (1998) in denoting market-level parameters with $\boldsymbol{\delta} = (\delta_{1m}^\mu, \delta_{2m}^\mu, \delta_{1m}^\gamma, \delta_{2m}^\gamma, \delta_{3m}^\gamma)$. ξ^μ and ξ^γ capture app-market specific idiosyncratic error terms. For example, a weather shock to market m may increase the marginal utility of Uber but not that of Google Docs. ε_{i1m} and ε_{i2m} are individual error terms that are iid across individuals but can be correlated across apps. ε_{i1m} and ε_{i2m} capture unobserved individual characteristics that affect utilities derived from apps. For example, users with cars, compared to those without cars, derive

[17] I thank one of the referees to point this out.

higher utilities from Google Maps and lower utilities from Uber. Therefore, the preference for Uber and the preference for Google Map can be negatively correlated. As discussed in Train (2009), the variance of μ_{ijm} cannot be separately identified from the mean of μ_{ijm}. I assume $(\varepsilon_{i1m}, \varepsilon_{i2m})$ follows a normal distribution $N(\mathbf{0}, \mathbf{\Sigma})$, where

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

ρ captures correlated preferences. As we add more controls in $\mathbf{x}_m$, ρ may be closer to zero. Given that we can never control for all relevant factors at the individual level, we should not assume $\rho = 0$. γ_{12} and ρ together explains the common user between app 1 and app 2. An econometric challenge is to disentangle γ_{12} from ρ, which will be discussed in Section §5.

3 Budget Competition and Functional Competition

In this section, I formally define budget competition and functional competition. I then provide an analytical characterization of budget competition in the quadratic utility framework. From the analytical characterization, budget competition can dominate functional competition and a pair of complementary apps can be gross substitutes. Lastly, I propose a model-free metric to gauge budget competition. This metric requires minimal data and can be used by industry analysts and antitrust authorities to quickly assess competition between apps.

3.1 Definition

When an app is shut down,[18] users will reallocate their time to the remaining apps. There are two reasons the exit of an app could affect other apps. First, because of substitutability (complementarity), users find the remaining apps more (less) appealing. Hence users will spend more (less) time on the remaining apps. This is the functional-competition effect. Second, the exit of an app means time that used to be spent on that app is now "free", and users can allocate it to the remaining apps. This is the budget-competition effect.

Consider the original bundle, $\mathbf{t}^o = \arg\max U(\mathbf{t})$, and the final bundle, $\mathbf{t}^f = \arg\max U(\mathbf{t}|\mu_j = -\infty)$, subject to the same time constraint $\sum_{k=0}^{J} t_k = T$. $\mathbf{t}^f - \mathbf{t}^o$ summarizes the effects of the exit of app j. To formally separate budget competition and functional competition, I introduce an intermediate step. In the intermediate step, the consumer chooses an intermediate bundle, $\mathbf{t}^i$, such that the marginal utilities of $\mathbf{t}^i$ equal to the marginal utilities of $\mathbf{t}^o$ except for app j. That is, $\mathbf{t}^i$ is the solution to the following system of equations:

[18]The following analysis applies to entry and price changes as well.

$$\frac{\partial U(\mathbf{t}^i|\mu_j = -\infty)}{\partial t^i_k} = \frac{\partial U(\mathbf{t}^o)}{\partial t^o_k} \quad \forall k \neq j \& t^i_k \geq 0 \tag{8}$$
$$t^i_j = 0.$$

Note that the time constraint is irrelevant in this step. $\mathbf{t}^i - \mathbf{t}^o$ is the functional-competition effect because the difference is entirely due to complementarity or substitutability among apps. $\mathbf{t}^f - \mathbf{t}^i$ is therefore the budget-competition effect. In the context of quadratic utility functions, we have a system of linear equations. If we have estimates of the parameters in the previous section, budget competition and functional competition are easy to calculate.

3.2 Analytical Characterization

In the model to be estimated, (8) can be reduced to one single equation when app 2 exits the market[19]:

$$\mu_1 + \gamma_1 t^o_1 + \gamma_{12} t^o_2 = \mu_1 + \gamma_1 t^i_1.$$

In Table 1, I provide general analytical solutions of functional competition and budget competition depending on if t^o_1 and t^i_1 are strictly positive. Note that for app 2 to have any competitive effect, be it budget competition or functional competition, t^o_2 must be strictly positive, which is implicitly assumed in Table 1. The results in Table 1 are intuitive. Let us focus on the first row and assume $\gamma_{12} \leq 0$, $t^o_1 > 0$ and $t^i_1 > 0$. In the intermediate step, $\frac{\gamma_{12}}{\gamma_1} t^o_2$ are diverted to app 1 due to functional competition. That leaves the user with $(1 - \frac{\gamma_{12}}{\gamma_1}) t^o_2$ of free time, which is allocated to the remaining options *proportional to the inverse of their satiation parameters*. The intuition is similar for complements. When $\gamma_{12} > 0$, t^i_1 decreases by $|\frac{\gamma_{12}}{\gamma_1}| t^o_2$. Therefore, the free time is $(1 + |\frac{\gamma_{12}}{\gamma_1}|) t^o_2$ and larger than t^o_2. For users with $t^o_1 > 0$ and $t^i_1 > 0$, the gross diversion ratio consists of two parts: diversion because of functional competition ($\frac{\gamma_{12}}{\gamma_1}$) and diversion because of budget competition ($(1 - \frac{\gamma_{12}}{\gamma_1}) \frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3}}$):

$$\text{Diversion Ratio} = \frac{\gamma_{12}}{\gamma_1} + (1 - \frac{\gamma_{12}}{\gamma_1}) \frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3}}. \tag{9}$$

From Table 1 and (9), budget competition is increasing in γ_{12} and γ_1. Another immediate implication of (9) is that budget competition can dominate functional competition and apps with a positive γ_{12} can be gross substitutes. How likely is this scenario? The answer depends on $\frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3}}$, which describes how free time is allocated to remaining options. It is a nonlinear function of deep parameters in a structural model. To calculate the size of

[19] The marginal utilities of app 3 and the offline option would not change with the exit of app 2 because I assume $\gamma_{20} = \gamma_{23} = 0$.

Table 1: Analytical Decomposition

t_1^o	t_1^i	Functional Competition ($t_1^i - t_1^o$)	Budget Competition ($t_1^f - t_1^i$)
$t_1^o > 0$	$t_1^i > 0$	$\frac{\gamma_{12}}{\gamma_1} t_2^o$	$(1 - \frac{\gamma_{12}}{\gamma_1}) t_2^o \frac{1}{\gamma_1(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3})}$
$t_1^o > 0$	$t_1^i = 0$	$-t_1^0$	$\max\{0, \frac{T - \frac{\mu_1-\mu_0}{\gamma_0} - \frac{\mu_1-\mu_3}{\gamma_3}}{\gamma_1(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3})}\}$
$t_1^o = 0$	$t_1^i > 0$	$\frac{\mu_2-\mu_1}{\gamma_1} + \frac{\gamma_2}{\gamma_1} t_2^o$	$[(1 - \frac{\gamma_2}{\gamma_1}) t_2^o - \frac{\mu_2-\mu_1}{\gamma_1}] \frac{1}{\gamma_1(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3})}$
$t_1^o = 0$	$t_1^i = 0$	0	$\max\{0, \frac{T - \frac{\mu_1-\mu_0}{\gamma_0} - \frac{\mu_1-\mu_3}{\gamma_3}}{\gamma_1(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3})}\}$

Note: This table presents analytical characterizations of functional competition and budget competition. The calculations are in Appendix B.

budget competition effect, we must estimate a full model as in Section §6. However, with some assumptions, we can transform $\frac{\frac{1}{\gamma_1}}{\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3}}$ into ratios of real-world variables.

3.3 A Model-Free Metric

I propose a model-free metric to gauge budget competition. To motivate this metric, consider two apps of interest, app 1 and app 2. Suppose we observe a user and her time spent on app 1 and app 2, t_1^* and t_2^*. The utility function of this user is

$$\begin{aligned} \max_{t_0, t_1, t_2 \geq 0} \; & t_0 + t_1 + t_2 - 0.0005 t_0^2 + \frac{1}{2}\gamma_1 t_1^2 + \frac{1}{2}\gamma_2 t_2^2 \\ s.t. \quad & t_0 + t_1 + t_2 = T \end{aligned} \tag{10}$$

where t_0 is the time spent on any other activities, online or offline. I assume $\mu_1 = \mu_2 = 1 = \mu_0$ and $\gamma_{12} = 0$ to avoid complicated estimation and obtain a closed-from metric. The solution to this maximization problem is $(t_0, t_1, t_2) = (T - t_1^* - t_2^*, t_1^*, t_2^*)$. We can solve for γ_1 and γ_2 with the FOCs. Consider the exit of app 2:

$$\begin{aligned} \max_{t_0, t_1 \geq 0} \; & t_0 + t_1 - 0.0005 t_0^2 + \frac{1}{2}\gamma_1 t_1^2 \\ s.t. \quad & t_0 + t_1 = T \end{aligned} \tag{11}$$

Let $(T - t_1', t_1')$ be the optimal time allocation when app 2 is absent. I prove in the appendix that

$$\Delta t_1 = t_1' - t_1^* = t_2^*(\frac{t_1^*}{t_0^* + t_1^*}) = t_2^*(\frac{t_1^*}{T - t_2^*}). \tag{12}$$

When app 2 exits the market, time that used to be spent on app 2 will be reallocated to the remaining apps *proportional to their time share before the exit.* $\frac{\frac{1}{\gamma_1}}{(\frac{1}{\gamma_0}+\frac{1}{\gamma_1}+\frac{1}{\gamma_3})}$ in our previous analysis becomes $\frac{t_1^*}{t_0^*+t_1^*} = \frac{t_1^*}{T-t_2^*}$.[20] The budget-competition effect of app 2 on app 1 is increasing in t_1^* and t_2^* if $t_2^* < 0.5T$. The numerator in (12) are the same for app 1 and app 2. Therefore, budget competition is largely symmetric between app 1 and app 2 unless $T - t_1 \gg T - t_2$ or the reverse. Adding more independent options (apps) would not change (12).

When we observe a group of users, the overall budget-competition effect is

$$\sum_i \Delta t_{i1} = \sum_i t_{i2}^*(\frac{t_{i1}^*}{T - t_{i2}^*}). \tag{13}$$

The total size of budget competition increases with the correlation between t_{i1}^* and t_{i2}^*. Alternatively, if we only observe aggregate usage of a set of markets, we can adopt a representative user approach and plug in average usage of app 1 and app 2 into (12).

Two assumptions are crucial to this metric. First, I assume all options to be independent to avoid estimating γ_{12} in a full model. Therefore this metric is more accurate when we expect $\frac{\gamma_{12}}{\gamma_1}$ to be close to zero. However, we can combine our belief of γ_{12} with the metric using (9). Second, I assume $\mu_1 = \mu_2 = 1 = \mu_0$ to have a closed-form metric of budget competition. Therefore, any difference in t_1 and t_2 is attributed to γ_1 and γ_2. This would be a serious concern if $s_1 \gg s_2$ or $s_1 \ll s_2$. In this case, we can focus on common users of app 1 and app 2 and $s_1 = s_2 = 1$ in this market. More generally, we can segment the market and apply the metric to sub-markets where s_1 and s_2 are similar. After estimating the full model, we can assess the usefulness of the metric by comparing the metric with results from the full model in Section §7.

4 Data

There are two types of app usage data available in the mobile Internet industry: individual level data and market level data. The first type of data resembles traditional surveys: firms pay individuals for their permission to install an app or software in order to monitor the usage of their devices. The data sets used by Han *et al.* (2016) , Lee (2018), Wu *et al.* (2022), and Boik *et al.* (2016) fall into this category. The data set used in this paper is aggregate market data, which is estimated based on a large quantity of observations from different sources. Wireless carriers and app developers are the two major sources. For

[20] t_3 merges into the outside option.

instance, China Unicom provides app usage data based on traffic data from its users.[21] App developers mostly use third-party libraries to analyze behaviors of their users.[22] Those data are then traded and matched based on unique device identifiers. In sum, market level data are estimated from snapshots of millions of devices, whereas individual data are 24×7 information from thousands of users. While both individual level data and market level data can be used to estimate relationships among apps, market level data are widely available in different countries and raise much less privacy concerns.[23]

Data used in this paper are from iResearch, a leading consulting firm in China with a focus on the mobile Internet industry. There are three parts of our data: the app usage data, the smartphone usage data, and the common user data. I introduce these data in the following subsections. All data are weekly data taken from the first quarter of 2017 in China.

4.1 App Usage Data

I acquired app usage data of the top 300 apps on Android cellphones of 290 demographic groups for 13 weeks in China. In this data set, a market is a demographic group defined by gender (male and female), age groups (below 24, 25-30, 31-35, 36-40, and above 40), and geographic areas (28 provinces and an "other" category). I do not have all the 300 apps' data as some apps have an estimated number of active users that is too small to be reliable. The threshold is 50,000. On average, I observe about 82 apps for each week-market pair. I observe more apps for large demographic groups in the data set. In total, I have 312,724 week-market-app observations. For each unit of observation, I observe the number of devices (per ten thousands) that used the app at least once during the week (henceforth, active user) and the average number of minutes spent on the app per device during the week (henceforth, average time spent). The summary statistics are in the upper panel of Table 2. The zeros in the table result from the technical difficulty in estimating usage of some apps (for example, input methods).

4.2 Smartphone Usage Data

iResearch provides total usage of Android devices, i.e., the smartphone usage data. Similarly, I have the number of active devices (per ten thousands) that are used at least once during the week (active users) and the average number of minutes spent on Android smartphones per device during the week (average time spent). With those data, I calculate market shares

[21] See https://www.cubigdata.cn

[22] For a story about how this works, see the report by the Wall Street Journal: https://www.wsj.com/articles/you-give-apps-sensitive-personal-information-then-they-tell-facebook-11550851636?mod=article_inline

[23] For example, Facebook shut down its "Facebook Research" app because of public anger. See https://www.wired.com/story/facebook-research-app-root-certificate

Table 2: Summary statistics of app usage

Variables	Mean	Min	Max	StdDev	# Obs	Unit
App Usage Data						
Active user	28.05	5	1074.2	46.38	312724	ten thousands
Market share	0.1064	0.004	0.958	0.132	312724	-
Average time spent	58.65	0	800.17	74.86	312724	minutes
Smartphone Usage Data						
Active user	225.05	10.31	1238.75	193.92	3770	ten thousands
Average time spent	1006	561.5	1435.5	202.08	3770	minutes
Common User Data						
Common user	599.21	6.41	29979.13	1387.733	79809	ten thousands

Note:
1, The smartphone and app usage data are weekly observations at the demographic group level from the first 13 weeks of 2017 in China. The common user data are weekly aggregate data for each pair of apps.
2. Active user of an app is the number of devices that used the app at least once during the week. Active user of smartphone is the number of Android smartphones that are used at least once during the week. Average time spent is the average number of minutes spent on the app per device during the week. Market share of an app is the active user of this app divided by the active user of Android smartphones in that market. Common user is the number of Android smartphones that use both apps at least once during the week.
3, The zeros in app usage data result from the technical difficulty of estimating usage of some apps, for example, input methods.
Data Source: iResearch.

of apps in each market which is the number of active users of an app divided by the number of active users of Android smartphones in that market. The summary statistics are in the middle panel of Table 2.

4.3 Common User Data

Most importantly, I have the common user data. For each pair of apps, I observe the number of Android smartphone users that used both apps at least once during the week (henceforth, common user). Again the 50,000 threshold applies. On average, I observe about 110 apps each week. I only have common user data at the national level because they are small, and hence unreliable, at the demographic group level. The summary statistics are in the lower panel of Table 2.

5 Estimation

5.1 Notation

I use GMM to match moments predicted by the model with moments calculated from the data. The full set of parameters to be estimated are $\boldsymbol{\theta} = (\boldsymbol{\beta}_1^{\mu}, \boldsymbol{\beta}_2^{\mu}, \boldsymbol{\beta}_1^{\gamma}, \boldsymbol{\beta}_2^{\gamma}, \boldsymbol{\beta}_3^{\gamma}, \gamma_{12}, \rho)$. As in

Nevo (1998), denote the linear parameters with $\boldsymbol{\theta}_1 = (\boldsymbol{\beta}_1^\mu, \boldsymbol{\beta}_2^\mu, \boldsymbol{\beta}_1^\gamma, \boldsymbol{\beta}_2^\gamma, \boldsymbol{\beta}_3^\gamma)$ as they will enter the GMM function linearly and the nonlinear parameters with $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$. I observe a set of markets, which are defined to be demographic groups, for 13 weeks. Denote weeks with w. For each market-week unit, I observe t_{1mw}^*, t_{2mw}^* and t_{3mw}^*, the average time spent on app 1, app 2, and all other apps in hours. For each week, I also observe the total number of common user between app 1 and app 2, c_{12w}^*. The asterisks indicate that they are observed variables. Hence the endogenous variables to be explained are $\mathbf{y}_{mw}^* = (s_{1mw}^*, s_{2mw}^*, t_{1mw}^*, t_{2mw}^*, t_{3mw}^*)$ and c_{12w}^*. The exogenous variables include $\mathbf{x}_{mw}$, a set of week and market fixed effects. Note that $\boldsymbol{\delta} = \mathbf{x}_{mw}\boldsymbol{\beta} + \boldsymbol{\xi}$.

With those notations, the model can be succinctly summarized as

$$(\mathbf{y}_{mw}^*, c_{12w}^*) = f(\boldsymbol{\delta}, \gamma_{12}, \rho) = f(\mathbf{x}_{mw}\boldsymbol{\beta} + \boldsymbol{\xi}, \gamma_{12}, \rho)$$

where $f(\cdot)$ is the nonlinear model described in Section §2 and $\boldsymbol{\xi}$ is the stack of all market level error terms. Note that there are five components in $\boldsymbol{y}_{mw}^*$ and five components in $\boldsymbol{\delta}$. At the market level, we have six outcome variables but seven parameters. The model is not identified with the observed variables we have.

5.2 Identification

The econometric challenge is to identify γ_{12} from ρ. Intuitively, both γ_{12} and ρ can explain c_{12w}^*. If one observes that many users use both NYTimes and WSJ, it could be the case that NYTimes and WSJ are complements as they offer different perspectives on the same events, or that users have a strong demand of news in general. In the first case, $\gamma_{12} > 0$. In the second case, $\rho > 0$. In economic textbooks, complements and substitutes are defined with compensated cross-price elasticities of demand: if an exogenous increase in the price of product A leads to a decrease in the compensated demand of product B, then they are complements; otherwise, they are substitutes. When there is no price, one can extend the definition: if users spend more time on an app due to an exogenous increase in its utility, the (marginal) utility of its complements (substitutes) would increase (decrease). This definition is based on cross derivatives of the utility functions and the basis of my identification strategy with updates as instruments. Updates of app 1 should change the *utility* of app 1 but not that of app 2. However, updates of app 1 can change the *usage* of app 2 through γ_{12}. Therefore, I use the following moments to identify nonlinear parameters γ_{12} and ρ

$$E(c_{12}^* - c_{12}) = 0 \tag{14}$$
$$E(update_{2w} \cdot \xi_{1mw}^\mu) = 0 \tag{15}$$
$$E(update_{1w} \cdot \xi_{2mw}^\mu) = 0 \tag{16}$$

The moment in (14) matches the observed common user and the predicted common user given γ_{12} and ρ. The moments in (15) and (16) are based on the assumption that the update history of app 1 (app 2) should not enter the utility of app 2 (app 1) directly. I use update history of the iOS version of the same app because update history of Android apps is not reliable.[24] Updates of the iOS version cannot possibly change utilities of any other Android app. Specifically, update history is described by three variables: the cumulative numbers of small updates, medium updates, and major updates.[25] Because I use cumulative number of updates, my IVs can capture the effects of updates even if users do not adopt updates immediately. As shown in the subscript of $update_{1w}$, this update history is common to all users in China and co-linear with time fixed effects. To circumvent this problem, I create market-specific update history variables, which allows each market to respond to the updates differently. Therefore, there are at most $3 \times M$ moments implied by (15). In Section §A, I provide reduced form evidence that the common user data and the update variables contain new information about the relationship between apps beyond the correlation of active users.

The identification of linear parameters $\boldsymbol{\beta}$ is straightforward and relies on the following moment conditions:

$$E(\mathbf{x}'_{mw}\xi^{\mu}_{1mw}) = 0 \tag{17}$$

$$E(\mathbf{x}'_{mw}\xi^{\mu}_{2mw}) = 0 \tag{18}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{1mw}) = 0 \tag{19}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{2mw}) = 0 \tag{20}$$

$$E(\mathbf{x}'_{mw}\xi^{\gamma}_{3mw}) = 0 \tag{21}$$

5.3 Implementation

Based on the above moments from (14) to (21), the GMM estimation is to minimize

$$\min_{\boldsymbol{\theta}} \boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi} + (c^*_{12} - c_{12})^2 \tag{22}$$

where $\boldsymbol{\xi}$ is the stack of all market level error terms and $\mathbf{z}_{mw} = (\mathbf{x}_{mw}, updata_{1w}, updata_{2w})$ collects all the exogenous variables. I separate $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ from $(c^*_{12w} - c_{12w})^2$ to highlight the fact that $\boldsymbol{\theta}_1$ enters $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ linearly and does not enter $(c^*_{12w} - c_{12w})^2$ given $\boldsymbol{\delta}$. Therefore, we can limit the globe search to $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$ as $\boldsymbol{\theta}_1$ is a linear function of $\boldsymbol{\delta}$.

This estimation follows Berry *et al.* (1995) with an inversion step and a global search step. I need to find the values of $\boldsymbol{\delta}$ that match the five observed market outcomes $\mathbf{y}^*_{mw} = (s^*_{1mw}, s^*_{2mw}, t^*_{1mw}, t^*_{2mw}, t^*_{3mw})$ given (γ_{12}, ρ). This is to solve the following system of non-

[24] One reason is that developers can publish Android apps outside mainstream app stores.

[25] "Small", "medium", and "major" are defined by the digits of version numbers.

linear equations,

$$\mathbf{y}^*_{mw} = \mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho). \tag{23}$$

Note that each component in $\mathbf{y}_{mw}$ is monotonically increasing in the corresponding component in $\boldsymbol{\delta}$. For example, given $(\delta^{\mu}_{2mw}, \delta^{\gamma}_{1mw}, \delta^{\gamma}_{2mw}, \delta^{\gamma}_{3mw})$ and (γ_{12}, ρ), s_{1mw} is increasing in δ^{μ}_{1mw}. I solve (23) by iterating on $\boldsymbol{\delta}$ analogously to the contraction mapping used by Berry *et al.* (1995) and Gowrisankaran & Rysman (2012):

$$\boldsymbol{\delta}^{new} = \boldsymbol{\delta}^{old} + \boldsymbol{\phi} \cdot \{ln(\mathbf{y}^*_{mw}) - ln(\mathbf{y}_{mw}(\boldsymbol{\delta}^{old}, \gamma_{12}, \rho))\} \tag{24}$$

where $\boldsymbol{\phi}$ are five positive tuning parameter used in the iterations.

Despite the appealing features of quadratic utility functions, there is no analytical solution to quadratic optimization problems. Therefore, I use numerical integration to form expectations of $\mathbf{y}_{mw}$. Let N_s be the number of simulations used for integration. We have

$$\mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho) = \frac{1}{N_s} \sum_{n=1}^{N_s} \mathbf{y}_{nmw}(\boldsymbol{\delta}, \gamma_{12}) \tag{25}$$

where $\mathbf{y}_{nmw}$ are the individual outcomes for the nth draw of $(\varepsilon_1, \varepsilon_2)$. In practice, I use 1000 Halton draws in the integration.

To summarize, the estimation consists of the following steps:

1. For a pair of (γ_{12}, ρ), invert out $\boldsymbol{\delta}(\gamma_{12}, \rho)$ with the mapping described in (24).

2. Calculate $c_{12}(\boldsymbol{\delta}(\gamma_{12}, \rho), \gamma_{12}, \rho)$ and $\boldsymbol{\xi}(\boldsymbol{\delta}(\gamma_{12}, \rho), \mathbf{z})$. Based on them, calculate the value of GMM function in (22).

3. Find (γ_{12}, ρ) that minimizes the GMM value calculated in step 2.

6 Estimation Results

I estimate the model on three representative pairs of apps to see how the model performs in different scenarios. For the first two pairs, I choose them because they are obviously a pair of substitutes (Baidu Map and Amap) and a pair of complements (Baidu and Baidu Map). A satisfactory model can infer the relationships from data. I study WeChat and Kwai because users spend a lot of time on them so that budget competition may be salient and the relationship between the two is *a priori* ambiguous. To reduce the computation burden, I aggregate market outcomes over provinces. Therefore, for each pair of apps, I have a panel of 11 markets[26] for 13 weeks.

[26] Gender and five age groups define 10 groups; and an "other" market to account for the difference between national usage and the total of balanced market level usage.

6.1 Substitutes

The first pair of apps are Baidu Map (app 1) and Amap (app 2), two dominant players in the mobile map market in China. During the 13 weeks, the number of active users of Baidu Map increased from 90 million to 110 million and that of Amap increased from 75 million to 100 million. The number of common users between Baidu Map and Amap increased from 11 million to 18 million. The summary statistics of market level variables are in Table 3.

Table 3: Summary Statistics of Baidu Map and Amap

Variables	Mean	StdDev	Min	Max	Unit
$s^*_{BaiduMap}$	0.1463	0.032	0.0877	0.2414	-
s^*_{Amap}	0.1277	0.0286	0.0832	0.2164	-
$t^*_{BaiduMap}$	0.0367	0.0085	0.0195	0.0652	hour
t^*_{Amap}	0.0746	0.0192	0.0421	0.1564	hour
t^*_{3mw}	16.6915	3.1601	10.5834	21.8704	hour

Note: $s^*_{BaiduMap}$ (s^*_{Amap}) is the number of active users of Baidu Map (Amap) divided by the number of active users of Android cellphones. $t^*_{BaiduMap}$ (t^*_{Amap}, t^*_{3mw}) is the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphone.
Data Source: iResearch.

The first three columns of Table 4 present the estimates of γ_{12} and ρ with different IVs. These estimates have the same sign and similar magnitudes. I use column (3) as my main results because both IVs are used. Baidu Map and Amap are estimated to be substitutes ($\hat{\gamma}_{12} = -1.15$), which confirms our prior belief. $\hat{\rho} = 0.7711$ suggests that Baidu map and Amap target the same group of users. Because the two apps offer similar functions, users who need Baidu Map will also find Amap useful. For the same reason, users who already use one would find the other redundant. A negative γ_{12} and a large ρ are what characterize a pair of direct competitors. For comparison, I also estimate γ_{12} with the assumption $\rho = 0$ in column (4) of Table 4. In this specification, Baidu Map and Amap are estimated to be almost independent apps. γ_{12} and ρ "substitute" each other in explaining the common user data: from column (3) to column (4), as ρ decreases from 0.7711 to 0, γ_{12} increases from -1.15 to -0.02.

γ_{12} is a structural parameter in a utility function. To understand the economic significance of γ_{12}, I quantify the value of substitutability/complementarity with compensating variation (CV). I calculate the compensating variations (CVs) of apps and pairs of apps. More specifically, I increase the total amount of time a user has to compensate for the loss of an app (or a pair of apps), such that the maximized utilities are the same before and after shutting down the app (the pair). The difference between the sum of CVs of each app and the CV of the pair is the value of substitutability/complementarity. This is the utility specification in the discrete model of Gentzkow (2007). This means that discrete choice models as in Gentzkow (2007) are a special case of my model. The CVs of Baidu

Table 4: Estimates for Baidu Map and Amap

	(1)	(2)	(3)	(4)
γ_{12}	-0.95	-1.15	-1.15	-0.02
	(0.0009)	(0.0003)	(0.0003)	(0.0015)
ρ	0.592	0.7711	0.7711	0
	(0.0001)	(0.0004)	(0.0004)	-
$update_1$ as IV	No	Yes	Yes	No
$update_2$ as IV	Yes	No	Yes	No

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
Data Source: The author's calculations.

Map, Amap, and the pair and the value of substitutability/complementarity are in Table 5. Substitutability is economically significant. Using the results in the last column in Table 5, the CV of Baidu Map is 9.4 hours for 1000 users when Amap is available. If Amap exits the market, consumers have one fewer option and the CV of Baidu Map would be 9.4+7.2=16.6 hours.

Table 5: Compensating Variations of Baidu Map and Amap

	(1)	(2)	(3)
CV of Baidu Map	10.7759	9.3945	9.3945
CV of Amap	18.6456	16.8407	16.8407
CV of Both	32.9073	33.4262	33.4262
Substitutability (Complementarity)	-3.4858	-7.1911	-7.1911
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-0.95,0.592)	(-1.15,0.7711)	(-1.15,0.7711)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The calculations are based on an anonymous market in the first week of 2017.
2, All numeric cells are the sum of CV in hours for all 1000 simulated users.
Data Source: The author's calculations.

6.2 Complements

The second pair of apps are Baidu (app 1) and Baidu Map (app 2). As the names suggest, they are developed by the same company, Baidu, Inc. The core functions of Baidu app are searching and news stream. I expect search engines and maps, and hence Baidu and Baidu Map, are complements. For example, when users search for locations, the first results often direct users to map apps. During the 13 weeks, the number of active users of Baidu fluctuated around 177 million. The number of common users between Baidu and Baidu Map increased from 30 million to 37 million. The summary statistics of market level variables are in Table 6. Note that there are slight differences between the summary statistics of s^*_{2mw} in Table 6 and the summary statistics of s^*_{1mw} in Table 3. This arises because the balanced panels of the two pairs are slightly different.

Table 6: Summary Statistics of Baidu and Baidu Map

Variables	Mean	StdDev	Min	Max	Unit
s^*_{Baidu}	0.2494	0.0464	0.1555	0.3321	-
$s^*_{BaiduMap}$	0.146	0.0323	0.0876	0.2414	-
t^*_{Baidu}	0.3086	0.0524	0.1475	0.4001	hour
$t^*_{BaiduMap}$	0.0366	0.0085	0.0195	0.0652	hour
t^*_{3mw}	16.4493	3.1086	10.4568	21.5881	hour

Note: s^*_{Baidu} ($s^*_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t^*_{Baidu} ($t^*_{BaiduMap}$, t^*_{3mw}) is the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphone.
Data Source: iResearch.

The estimates of (γ_{12}, ρ) are in Table 7. The coefficients in the first column are quite different from those in the other two columns. This suggests that the IV in column (1) is not ideal. As before, I treat results in column (3) as the main results. The main results confirm our prior belief that Baidu and Baidu Map are complements. As in the previous subsection, I calculate the CVs of Baidu Map, Amap, and the pair and the value of substitutability/complementarity in Table 8. The CVs of Baidu Map are in line with those in Table 5. The complementarity is small in magnitude but large as a percentage of the CV of Baidu Map for columns (2) and (3). This suggests that Baidu Map relies on Baidu but not the reverse. Baidu has other apps named after Baidu and they are in the "Baidu Core" business unit. It is reasonable to assume that those apps are also complements with the Baidu app.

Table 7: Estimates for Baidu and Baidu Map

	(1)	(2)	(3)
γ_{12}	-0.4613	0.296	0.1467
	(0.0006)	(0.0035)	(0.0005)
ρ	0.5522	-0.1642	-0.0448
	(0.0006)	(0.0001)	(0.0005)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
Data Source: The author's calculations.

Table 8: Compensating Variations of Baidu and Baidu Map

	(1)	(2)	(3)
CV of Baidu	189.5948	193.6434	194.2896
CV of Baidu Map	10.2916	10.8814	11.1509
CV of Both	210.6131	202.3202	204.2014
Substitutability (Complementarity)	-10.7266	2.2045	1.239
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-0.4613,0.5522)	(0.296,-0.1642)	(0.1467,-0.0448)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The calculations are based on an anonymous market in the first week of 2017.
2, All numeric cells are the sum of CV in hours for all 1000 simulated users.
Data Source: The author's calculations.

6.3 Independent Apps

The last pair of apps I studied are WeChat (app 1) and Kwai (app 2). WeChat is the flagship app of Tencent, which was first released in 2011. By the first quarter of 2017, the main functions include instant messaging, social media ("Moments"), mobile payment ("WeChat Pay"), content distribution ("Subscriptions"), and app store ("mini program"). It is a super-app used by almost all smartphone users in China. From Table 9, users spent about one fourth of their smartphone time on WeChat. Given it market dominance, WeChat is competing with all other apps for user time. Kwai is a video-sharing app that features short videos and live-streaming. Thanks to the recommendation algorithms, short video apps like Kwai and Tik Tok are often referred to as "a black hole of time". In terms of their functions, WeChat and Kwai seems to be independent or weak substitutes in the broad sense of social networking. However, WeChat and Kwai share a lot of common users. During the 13 weeks, the number of active users of WeChat fluctuated around 555 million, and those of Kwai increased from 78 million to 81 million. The number of common users between WeChat and Kwai is about 70 million. It is tempting to conjecture that the two apps are complements based on the number of common users. Overall, the competitive relationship between WeChat and Kwai is ambiguous.

The estimates of (γ_{12}, ρ) are in Table 10. As before, I treat results in column (3) as the main results. $\hat{\gamma}_{12} = -0.08$ refutes the conjecture that WeChat and Kwai are complements. The large number of common users is explained by the positive correlation between the preference for WeChat and that for Kwai ($\hat{\rho} = 0.42$). This suggests that the "budget-competition" effect of one app on the other may be large. With these estimates, I calculate the CVs of WeChat, Kwai, and the pair and the value of substitutability/complementarity in Table 11. When WeChat is shut down, the CV of Kwai would increase by more than 150%

Table 9: Summary Statistics of WeChat and Kwai

Variables	Mean	StdDev	Min	Max	Unit
s^*_{WeChat}	0.8352	0.0532	0.731	0.9346	-
s^*_{Kwai}	0.1154	0.0148	0.0864	0.1451	-
t^*_{WeChat}	4.3685	0.5473	3.2078	5.5869	hour
t^*_{Kwai}	0.1896	0.017	0.1519	0.2246	hour
t^*_{3mw}	12.2633	2.6741	6.7176	16.3154	hour

Note: s^*_{WeChat} (s^*_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t^*_{WeChat} (t^*_{Kwai}, t^*_{3mw}) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphone.
Data Source: iResearch.

in column (3). If Kwai is shut down, the CV of WeChat would also increase significantly. WeChat and Kwai are competing for user time despite a large number of common users and seemingly independent functions. Note that this conclusion is drawn based on data from 2017. After failed attempts to promote its own short-video app WeSee, Tencent invested \$2 billion in Kwai in December 2019[27], and is now the largest institutional investor of Kwai after its IPO. Tencent also added short-video function in WeChat in the first quarter of 2020 to directly compete with Kwai and Tik Tok.

Table 10: Estimates for WeChat and Kwai

	(1)	(2)	(3)
γ_{12}	-0.14	-0.02	-0.08
	(0.0001)	(0.0001)	(0.0015)
ρ	0.76	0.18	0.42
	(0.0001)	(0.0001)	(0.0024)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
Data Source: The author's calculations.

6.4 Discussion

Comparing the results of the three pairs of apps, the substitutability/complementarity term is positive if and only if γ_{12} is positive. Estimated γ_{12} cannot be compared across models. $\hat{\gamma}_{12} = -1.15$ in column (3) of Tables 5 and $\hat{\gamma}_{12} = -0.08$ in column (3) of Table 11. By contrast, the substitutability/complementarity term is -7.2 hours in the first case and -154.9 hours in the second case.

[27]See https://www.scmp.com/tech/apps-social/article/3041747/tencent-said-invest-us2-billion-short-video-app-kuaishou

Table 11: Compensating Variations of WeChat and Kwai

	(1)	(2)	(3)
CV of WeChat	3191.5925	3635.2672	3567.6392
CV of Kwai	70.0809	102.6322	94.9661
CV of Both	3853.5602	3765.7854	3817.5046
Substitutability (Complementarity)	-591.8868	-27.886	-154.8994
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-0.14,0.76)	(-0.02,0.18)	(-0.08,0.42)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, The calculations are based on an anonymous market in the first week of 2017.
2, All numeric cells are the sum of CV in hours for all 1000 simulated users.
Data Source: The author's calculations.

In the above tables, I report only estimates of γ_{12} and ρ and coefficients of other covariates (fixed effects) are omitted because there are many fixed effects. In Section §B, I report week fixed effects and aggregate market fixed effects to demographic levels for Baidu Map and Amap. Male users and users aged between 31-35 derive higher utility from Baidu Map and Amap because they are more likely to own a car in China.

The demand model can further incorporate other notable features in the mobile Internet industry (for example, advertisement and two-sidedness). In an earlier version of this paper, I estimate a similar model with network effects by adding active users of app 1 in a reference market in μ_{1mw} (Weiergraeber, 2022). The estimation results are similar.[28]

7 Budget Competition Revisited

In this section, I revisit budget competition with the estimated full model. To better understand the competitive relationship between apps, I shut down one of the two apps to see how the usage of the other app would change. I calculate diversion ratios based on the counterfactual results: if one app exits the market, how much of its usage will be diverted to another app?[29] With these simulations, app developers would know who competes time away from their apps. I then decompose the competitive effects of one app on another into "functional competition" and "budget competition" according to the definition in Section §3. I compare the decomposition results with the model-free metric. Despite the simplifying assumptions of the metric, the results are close. Lastly, I use several examples to discuss

[28]The results are available upon request.

[29]Diversion ratios are an important tool of antitrust authorities to analyze horizontal mergers. In the 2010 Horizontal Merger Guidelines, " Diversion ratios between products sold by one merging firm and products sold by the other merging firm can be very informative for assessing unilateral price effects, with higher diversion ratios indicating a greater likelihood of such effects."

how we can combine the metric with other institutional knowledge to more accurately assess budget competition.

7.1 Competitive Effects

In this subsection, I simulate counter-factuals in which one of the apps is shut down. For each pair of apps, I simulate market outcomes for an anonymous market in the first week of 2017 with different sets of (γ_{12}, ρ).[30] In Table 12, columns (2) and (3) present counter-factuals for the baseline estimates and columns (4) and (5) present counter-factuals for the estimates in the last column of Table 4 where we assume $\rho = 0$.

Table 12: Counter-factuals of Baidu Map and Amap

	Observed	Baseline		Assume $\rho = 0$	
	Outcomes (1)	No Baidu Map (2)	No Amap (3)	No Baidu Map (4)	No Amap (5)
$s_{BaiduMap}$	0.1056	0	0.139	0	0.107
s_{Amap}	0.0832	0.103	0	0.084	0
$t_{BaiduMap}$	0.0224	0	0.0314	0	0.022
t_{Amap}	0.0421	0.0497	0	0.0426	0
t_3	15.559	15.559	15.56	15.559	15.56
Diversion Ratio	-	35.04%	21.28%	0.18%	0.13%
$(\hat{\gamma_{12}}, \hat{\rho})$	-	(-1.15,0.7711)		(-0.02,0)	

Note:
1, The observed outcomes in column (1) are from an anonymous market in the first week of 2017.
2, $s_{BaiduMap}$ (s_{Amap}) are the number of active users of Baidu Map (Amap) divided by the number of active users of android cellphones. $t_{BaiduMap}$ (t_{Amap}, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphones.
3, Diversion ratio is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

The two sets of simulated outcomes are vastly different. When $\gamma_{12} = -0.02$ and $\rho = 0$, shutting down one app has almost no effect on the other. By contrast, when $\gamma_{12} = -1.15$ and $\rho = 0.7711$, the market share of Baidu Map would increase by 3.34 percentage points if I shut down Amap, and the market share of Amap would increase by 2 percentage points if I shut down Baidu Map. Consider the case of shutting down Amap when $(\gamma_{12}, \rho) = (-1.15, 0.7711)$. The inversion process reveals that there are 1.4% users that use both Baidu Map and Amap. Therefore, there are 6.92% users using Amap but not Baidu Map. When Amap is shut down, 3.34% out of the 6.92% unique users turn to Baidu Map. This implies an diversion ratio (in terms of unique active user) of $\frac{3.34}{6.92} = 48.3\%$. When we focus on time spent, diversion ratio

[30]Specifically, I invert out $\boldsymbol{\delta}$ for the two pairs of (γ_{12}, ρ) and then set δ^{μ}_{2mw} (δ^{μ}_{1mw}), the mean marginal utilities of app 2 (app 1), to be a very small number, -20, and simulate the market outcomes.

is simply the increase in the total time spent on Baidu Map divided by the total time spent on Amap before its exit: $\frac{\Delta t_1}{t_2} = \frac{0.0314-0.0224}{0.0421} = 21.28\%$. In other words, when Amap exits the market, 21% of its time goes to Baidu Map and 79% of its time goes to offline activities and the generic app. Note that the effects of shutting down Amap on Baidu Map are larger than the reverse. When Baidu Map is not available, the market share of Amap would only increase by 2 percentage points.

The counter-factual results for Baidu and Baidu Map with the baseline estimates are in Table 13. Shutting down Baidu Map has negligible effects on Baidu.[31] However, shutting down Baidu would reduce the market share of Baidu Map by 0.86 percentage point and the time spent on Baidu Map by more than 10% of the observed level. It is therefore no surprise that the company Baidu treats the Baidu app as its core business.[32] A caveat to my findings is that the discovery process of apps is not modeled in this paper. Cross-promotion between apps developed by the same company is a widely used marketing strategy.[33] Promoting Baidu Map with Baidu will lead to a persistent large number of common users if there are significant switching costs. Diversion ratio is -10% when Baidu Map exits the market. Therefore, even if Baidu Map is unprofitable, Baidu, Inc. may choose to maintain it's operation due to its contribution to the usage of the Baidu app. This is why tech conglomerates often develop and operate unprofitable apps for extended periods.

Table 13: Counter-factuals of Baidu and Baidu Map

	Observed Outcomes (1)	No Baidu (2)	No Baidu Map (3)
s_{Baidu}	0.2459	0	0.259
$s_{BaiduMap}$	0.1056	0.097	0
t_{Baidu}	0.3222	0	0.3201
$t_{BaiduMap}$	0.0225	0.0201	0
t_3	15.279	15.284	15.28
Diversion Ratio	-	-0.74%	-10%
$(\hat{\gamma}_{12}, \hat{\rho})$		(0.1467,-0.0448)	

Note:
1, The observed outcomes in column (1) are from an anonymous market in the first week of 2017.
2, s_{Baidu} ($s_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t_{Baidu} ($t_{BaiduMap}$, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphones.
3, Diversion ratio is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

[31] The difference between 0.263 and 0.26 is due to simulation error.

[32] In the annual reports of Baidu, Inc., Baidu app, Baidu Map and other apps named after Baidu are in the "Baidu Core" business group.

[33] See The Ultimate Mobile Marketing Playbook by App Annie at https://www.appannie.com/en/insights/aso-app-store-optimization/ultimate-mobile-marketing-playbook/

The counter-factuals for WeChat and Kwai with the baseline estimates are in Table 14. As expected, the competitive effects of WeChat on Kwai are larger than the reverse. Given that WeChat is the dominant player and Kwai is the entrant, it would be more interesting to focus on the effect of Kwai on WeChat. The market share of WeChat does not change in response to the exit of Kwai. This is because almost all Kwai users (11.7% out of 12.2%) already use WeChat. A diversion ratio of 29% means that about 29% of the time spent on Kwai comes from WeChat. The remaining 71% mostly comes from offline activities.[34] 0.06 (3.8-3.74) hours seem to be small for a representative Android smartphone user. For the 122 users who use Kwai among the 1000 simulations, their time spent on WeChat would increase on average by about 30 minutes ($\frac{0.06}{12.2\%} \approx 0.5$ hour) if Kwai is shut down. For WeChat, competition from Kwai is significant. A caveat is that the competitive effects of Kwai on WeChat I estimated with data from 2017 is likely to be a lower bound. Usage of WeChat and that of Kwai have grown significantly since then. Tencent also added short-video and live streaming to WeChat to directly compete with Kwai and Tik Tok in the first quarter of 2020. The competitive effects will be much larger now.

Table 14: Counter-factuals of WeChat and Kwai

	Observed Outcomes (1)	No WeChat (2)	No Kwai (3)
s_{WeChat}	0.7725	0	0.779
s_{Kwai}	0.1221	0.222	0
t_{WeChat}	3.7391	0	3.7984
t_{Kwai}	0.2025	0.4257	0
t_3	11.6821	11.7173	11.6835
Diversion Ratio	-	5.98%	29.08%
$(\hat{\gamma}_{12}, \hat{\rho})$	-	(-0.08,0.42)	

Note:
1, The observed outcomes in column (1) are from an anonymous market in the first week of 2017.
2, s_{WeChat} (s_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t_{WeChat} (t_{Kwai}, t_3) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphones.
3, Diversion ratio is the increase in the time spent on the remaining app divided by the time spent on the exit app before its exit.
Data Source: The author's calculations.

7.2 Decomposition

In Table 15, I decompose the competition effects of one app on another using the definition in section 3.1 and compare the decomposition results with the model-free metric proposed in section 3.3. We would have more confidence in the metric if the metric and the decomposition results calculated from the full model are close.

[34] I assume away complementarity and correlated preference between app 1 (app 2) and the generic app.

Table 15: Functional Competition and Budget Competition

	Baidu Map and Amap	Baidu and Baidu Map	WeChat and Kwai
The Exit of App 1			
Budget Competition	0.0027	0.0126	3.3248
Functional Competition	7.875	-2.4075	220.1286
Total Effects on App 2	7.8777	-2.3949	223.4533
The Model-Free Metric	0.0056	0.0432	4.6095
The Exit of App 2			
Budget Competition	0.0091	0.0191	0.5007
Functional Competition	8.8863	-2.2601	58.2947
Total Effects on App 1	8.8954	-2.2409	58.7954
The Model-Free Metric	0.0056	0.0432	4.5124
t_1^*	0.0224	0.3222	3.7391
t_2^*	0.0421	0.0225	0.2025
estimates $(\hat{\gamma}_{12}, \hat{\rho})$	(-1.15,0.7711)	(0.1467,-0.0448)	(-0.08,0.42)

Notes:
1, This table is based on data from an anonymous market in the first week of 2017.
2, The cells corresponding to the model-free metric is calculated according to (12) and then times 1000 so that they are comparable to results from the full model.

As shown in Table 15, the metric produces estimates reasonably close to the results from the full model despite the simplifying assumptions. One exception is the budget competition of Kwai on WeChat. The metric (4.5) is 9 time the decomposition from the model (0.5). The key reason is that the market share of WeChat is more than 6 times that of Kwai, which violates the assumption that $\mu_1 = \mu_2$. This is less of a concern now because of the spectacular growth of Kwai since 2017.

7.3 Usefulness of the Metric

In this section, I discuss several examples to show how this metric can be useful. The examples will show that the assumptions in section 3.3 are not as restrictive as they may seem and the metric can be combined with institutional knowledge to more accurately gauge budget competition.

Let us first continue the discussion of WeChat and Kwai in section 7.2. Time spent on WeChat and Kwai and their market shares increased significantly since 2017. $\mu_1 = \mu_2$ is a more realistic assumption in the current market. A reasonable guess is that for all smartphone users in China, $t_1^* = 20$ and $t_2^* = 10$ for a typical week in 2023.[35] If Kwai exits the market, the budget-competition effect on WeChat is $10 \times \frac{20}{168-10} = 1.27$ for each one of the about 1 billion smartphone users. The gross diversion ratio in (9) can be positive for a pair of complements. For the sake of argument, let us assume $\frac{\gamma_{12}}{\gamma_1} = -0.05$ for WeChat and Kwai and we approximate $\frac{1}{\gamma_1} \frac{1}{\frac{1}{\gamma_0} + \frac{1}{\gamma_1} + \frac{1}{\gamma_3}}$ with $\frac{20}{168-10} = 0.127$. The gross diversion ratio

[35] See the report by QuestMobile at https://www.questmobile.com.cn/research/report/1686624886410285058

is $-0.05 + 1.05 \times 0.127 = 0.083$. Therefore, a pair of complementary apps can be gross substitutes. Another way to use (9) is to calculate the threshold of $\frac{\gamma_{12}}{\gamma_1}$ above which the two apps are gross substitutes. In this case, the threshold is $\frac{-0.127}{1-0.127} = -0.145$. In other words, the diversion ratio implied by complementarity must be larger than 14.5% for WeChat and Kwai to be gross complements. This is highly improbable for WeChat and Kwai. A pair of complementary apps can be gross substitutes because of budget competition. Budget competition implies that being too "large" *per se* is a source of antitrust concern when analyzing mergers of apps.

Netflix CEO Reed Hastings said in 2017 that "We're competing with sleep" (Hern, 2017). Using this metric, we can put a ballpark figure on this claim. Assume Netflix users spend 0.5 hours per day on Netflix[36] and 7 hours on sleep. Then the budget competition effect of Netflix on sleep is $\frac{0.5\times 7}{24-0.5} \approx 0.15$, which is about 9 minutes per day. This did not take into account the binge-watching habit of Netflix users. On a binge-watching day, assume a user spend 4 hours on Netflix and 5 hours on sleep. The budget competition effect of Netflix on sleep is larger: $\frac{4\times 5}{24-4} = 1$. In other words, if she does not binge-watch, her sleep would increase by 1 hour due to budget competition. This is more reasonable than 9 minutes. We can do even better. Recall the quote from Reed Hastings: "Think about if you didn't watch Netflix last night: What did you do? There's such a broad range of things that you did to relax and unwind, hang out, and connect–and we compete with all of that."(Raphael, 2017) Assume a user spend 12 hours on work and related activities and she can only watch Netflix/sleep/connect in the remaining 12 hours. In this case, budget competition is $\frac{4\times 5}{12-4} = 2.5$. This number should be closer to what Reed Hastings had in mind when he said "We're competing with sleep". This is essentially a multi-stage budgeting model. As shown in the Netflix example and the WeChat/Kwai example, this metric can be combined with institutional knowledge ($\frac{\gamma_{12}}{\gamma_1}$, binge-watching, multi-stage budgeting) to more accurately gauge budget competition and assess overall competition.

Rather than taking the metric at face value, we can use this metric to gauge the order of magnitude of budget competition, which is often exaggerated or downplayed in business and legal settings. In the high-profile antitrust lawsuit filed by Qihu against Tencent in 2012, Tencent was accused of abusing its market dominance in the instant messaging market, wherein its software QQ had a market share of 80%–95% according to different measures. Tencent countered that the relevant market should include virtually all Internet companies and their software because they were all competing for user time.[37] Tencent exaggerated budget competition to obfuscate its market dominance in the instant messaging market. With this metric and some aggregate usage data, we can easily dismiss this argument.

Both Wu (2018) and Hughes (2019) called for the breakup of Facebook and Instagram. What is the role of budget competition in the Facebook-Instagram case? According to data

[36] See the estimates by eMarketer at https://www.insiderintelligence.com/chart/232130.

[37] The court did not accept this market definition and stick to the market definition based on functions.

from eMarketer[38], we can assume each day an average US adult spend one hour on Tik Tok and YouTube respectively, and 0.5 hour on Twitter, Instagram, Snapchat, Facebook, and Reddit respectively in June 2023. The total time budget on social media is assumed to be 5 hours. Then if Instagram is shut down by Meta, the budget competition effect on Facebook is $0.5 \times \frac{0.5}{5-0.5} = 0.056$ hour, which is about 3.3 minutes per day. The diversion ratio implied by budget competition is $\frac{0.5}{5-0.5} = 11.1\%$. Budget competition is non-negligible and should be incorporated into antitrust discussions of the Facebook-Instagram case. The budget competition effect is much larger between Tik Tok and YouTube ($1 \times \frac{1}{5-1} = 0.25$) because users spend much more time on them. The effect will be larger if the growth trend of Tik Tok and YouTube continues. YouTube rolled out "Shorts", its short video function, to directly compete with Tik Tok in the fall of 2020.

8 Supply Side and Mergers

In the previous section, I quantify competition with counter-factuals and diversion ratios. Competition is then decomposed into functional competition and budget competition. However, we still do not know how competition, or the lack of it, can affect consumer welfare. In other words, how can consumers be hurt if prices of apps are zero anyway? In this section, I combine the estimated demand model with a supply model. With the demand and supply model, I propose a different version of the hypothetical monopolist test and simulate mergers of the three pairs of apps.

8.1 Price and the Supply Side

To evaluate the effects of a merger of apps on firms and consumers, we need firms to have a decision variable which affects both consumer demand and profit. Traditionally, price is the decision variable. However, most apps are free to use. App developers make profits by selling user attention to advertisers (Prat & Valletti, 2022). Users dislike prices and ads whereas firms prefer higher prices and ad load whenever possible. Some developers adopt a freemium strategy: they offer a free version with lower quality (higher ad load or fewer features) and a premium version with higher quality (lower ad load or more features). Conceptually, price, ad load, and quality are isomorphic in demand and supply models of apps unless we add more features to the model: transaction costs, adjustment costs, differential curvatures of ad and money in user preference, etc.[39] In the following model, I still refer to the decision variable as "price" on the understanding that it can be ad load or quality.

[38] See the details at https://www.insiderintelligence.com/chart/263759/average-time-spent-per-day-by-us-adult-users-on-select-social-media-platforms-2023-minutes.

[39] For example, it is more costly to collect small payments from millions of users than to transact with a few advertisers. This is why most developers choose not to charge users directly.

To simulate mergers of free apps, I need to specify the profit function and how money enters the utility function. For simplicity, I assume the utility function is linear in money[40]:

$$\max_{t_{i0},t_{i1},t_{i2},t_{i3}\geq 0} U(\mathbf{t}) - \alpha(p_1 t_{i1} + p_2 t_{i2}) \tag{26}$$
$$s.t. \quad t_{i0} + t_{i1} + t_{i2} + t_{i3} = 168$$

where $U(\mathbf{t})$ is from (2). p_1 and p_2 are the hourly prices of app 1 and app 2. Note that in (26), the marginal value of time in terms of money is $\frac{1-0.001t_0^*}{\alpha}$. This ratio is estimated to be \$1.65 per hour for Taiwan in 2001(Shiaw, 2004). Therefore, I assume $\frac{1}{\alpha} = 8.5$ yuan.[41] This number would not change any qualitative results. We can also have different αs for different demographic groups and maximize the aggregate profits.

The profit function of a monopolist with an app $j = 1, 2$ is

$$\Pi_j = p_j \sum_i t_{ij} + r_j \sum_i t_{ij} - \Psi \tag{27}$$

where Ψ is the fixed cost and r_j is the advertising revenue per hour. If we expand $U(\mathbf{t})$ in (26), the "net marginal utility" is now $\mu_j - \alpha P_j$. Therefore, a price change is equivalent to a change in μ_j for consumers. If consumers dislike advertisements, an increase in price is equivalent to an increase in ad load, which reduces μ_j. In (27), $p_j + r_j$ is the total revenue per hour. Because ad load is implicitly in both r_j and μ_j, a price change is equivalent to a change in r_j for firms. In later discussions, $p_j + r_j$ is the "comprehensive price". Overall, an increase in the price of an app is equivalent to a decrease in quality or an increase in ad load in the demand and supply of apps.

All the three components in the profit functions can change after a merger. If there are cost synergies, Ψ would change. Prat & Valletti (2022) are concerned about the increased market power in the advertising market after a merger, and r_{jm} would change in that case. In the following analysis, I assume $\Psi = 0$. I then back out r_j from profit maximization conditions and keep it fixed so that I can focus on how p_j would change after a merger. In other words, the post-merger monopolist does not have market power in the advertising market and exercise its market power in the app market by changing p_j.[42]

With (27), I find the smallest r_j with which the profit-maximizing price is zero. It is straightforward to find such r_j except for the pair of Baidu and Baidu Map because

[40] This is a reasonable assumption given that users are unlikely to spend a significant share of monetary wealth on apps.

[41] Considering the GDP per capita of the Taiwan province in 2001 and that of Mainland China in 2017, a comparable estimate for $\frac{1-0.001t_0^*}{\alpha}$ in our context is 7.2322 yuan. As the average of t_0 is 16.76 hours, $\frac{1-0.001t_0^*}{\alpha} \approx \frac{0.85}{\alpha} = 7.2322$. Therefore, $\frac{1}{\alpha} = 8.5$ yuan.

[42] To be more precise, r_j can change if either ad load or the unit price of ad changes. A change in p_j can be seen as a change in ad load in the current model. However, we need to explicitly model the advertising market to analyze a change in the unit price of ad.

they are owned by the same company, Baidu, Inc. In this case, I consider the joint profit maximization problem of Baidu and Baidu Map. I find the pair of $(r_{Baidu}, r_{BaiduMap})$, with which the profit-maximizing prices, $(p_{Baidu}, p_{BaiduMap})$, are zero and $r_{Baidu} + r_{BaiduMap}$ is the smallest. If firms deviate from (static) profit maximization, I would underestimate or overestimate r_j. Firms may set a low price to capture the market and set a high price later to harvest due to dynamic reasons like inertia, addiction, and network effects. In this case, I will overestimate r_j. A biased r_j could lead to "wrongs" prices, profits, and consumer surplus. However, because I keep r_j fixed in the merger simulations, bias in r_j would not change our qualitative conclusions.

8.2 The Monopolist Test

An implication of (27) is that if r_j is sufficiently large, p_j should be negative to maximize profits. A price of zero is one of the infinitely many prices firms can choose from and not special at all.[43] Traditional antitrust analysis cannot handle zero pricing well. Consider the widely used monopolist test, what is a "Small but Significant and Non-transitory Increase in Price" (SSNIP) if the observed prices are zero? In the landmark case of Qihoo v. Tencent, the People's Supreme Court held that SNNIP is inappropriate when observed prices are zero and suggested a different version of the monopolist test using a "Small but Significant and Non-transitory Decrease in Quality" (SSNDQ).[44] From (27), we should consider a percentage change in $p_j + r_j$ rather than p_j. To facilitate discussion, I refer to it as a "Small but Significant and Non-transitory Increase in Comprehensive Price" (SSNICP). In this version of the monopolist test, I consider a monopolist who own the relevant apps and increase the comprehensive prices of all apps, $p_j + r_j$, by 5%. I then check whether the total profits increase. According to the simulation results, Baidu Map and Amap constitute a relevant market whereas WeChat and Kwai does not. While there are other map apps like Tencent Map, they are too small to constrain the market power of Baidu Map and Amap. As an example, for one market in my data, Baidu Map and Amap had more than one million active users and Tencent Map had about 56,000 active users. Considering that users spent about one fourth of their smartphone time on WeChat, WeChat was likely exercising monopoly market at the time and a 5% increase in the comprehensive price would be costly. The results are consistent with merger simulations in the next subsection.

[43] In the report commissioned by the Stigler Committee on Digital Platforms(Scott Morton *et al.*, 2019):

> "Free" is not a special zone where economics or antitrust do not apply. Rather, a free good is one where the seller has chosen to set a monetary price of zero and may set other, non-monetary, conditions or duties. It is possible that a digital market has an equilibrium price that is negative; in other words, because of the value of target advertising, the consumer's data is so valuable that the platform would pay for it.

[44] See https://www.lexology.com/library/detail.aspx?g=1742654e-8b5c-4197-bafc-f267fd78b3e8 for an introduction.

8.3 Mergers

The merger simulation results are in Table 16. I simulate 1000 users with parameters estimated for an anonymous market in the first week of 2017. Ad revenue per hour implied by the profit maximization conditions are in the top row. An hour on WeChat or Baidu is more valuable than an hour on Baidu Map/Amap/Kwai. Baidu users use the search engine with specific needs, which makes them valuable to advertisers with corresponding services and products. Super apps like WeChat also know a lot about its users through their many features. Because of WeChat Pay, the conversion rate of WeChat users may also be higher. Both would make WeChat users valuable to advertisers.

The market outcomes before and after a merger of Baidu Map and Amap are in column (1). The results are intuitive for a pair of substitutes: prices and profits increase and consumer surplus decreases. The comprehensive price ($p_j + r_j$) of Baidu Map increases by $\frac{1.16}{3.45+0} = 33.6\%$ and that of Amap increases by $\frac{0.465}{3.78+0} = 12.3\%$. As with other mergers of direct competitors, the loss in consumer surplus (41 yuan) is much larger than the gain in profits (5.5 yuan). Such mergers should be blocked if antitrust agencies want to maximize total surplus. The merger results are consistent with the SSNICP conclusion.

In column (2), the post-merger results are the observed market outcomes because Baidu and Baidu Map are owned by the same company, Baidu, Inc.. The pre-merger results are the counterfactual. If Baidu, Inc. divests itself of Baidu Map, the comprehensive price of Baidu will increase by 0.021 (0.4%) and that of Baidu Map by 0.401 (11%). Baidu, Inc. internalizes the complementarity between Baidu and Baidu Map and set lower comprehensive prices. For 1000 simulated users in one week, the consumer surplus increases by 15 yuan and the profits increase by 0.5 yuan because of the common ownership. The results in column (2) suggest that developing complementary apps around a flagship app can be a profitable strategy for tech firms. Baidu, Inc. has other apps named after Baidu (for example, Baidu Browser and Baidu Netdisk). The "Baidu Core" in Baidu's financial reports consists of those complementary apps and notably Baidu.com.[45] The overall complementarity between Baidu and its satellite apps can be large.

We can compare profits calculated in column (2) in Table 16 with actual data in Baidu's financial reports. The total revenue from Baidu and Baidu Map is about 1700 yuan for 1000 smartphone users in one week. The implied revenue is about 75.14 billion RMB for the year of 2017.[46] The actual revenue of the Baidu Core in 2017 is 67.7 billion RMB. The two number are of similar magnitude. Considering that revenue from baidu.com and other apps can be significant, there are significant upward bias in the recovered r_j and hence the profits in column (2) in Table 16. There are several reasons for the discrepancies. First,

[45] For a detailed explanation of "Baidu Core", see the Form 20-Fs of Baidu, Inc.

[46] According to the report by QuestMobile, the total number of monthly active user of smartphone is about 1 billion. Assume that the total number of weekly active user of smartphone is 850 million in 2017. $1700 \times 52 \div 1000 \times 0.85$=75.14 billion.

this anonymous group of users derive higher utilities from Baidu and Baidu Map than many other groups. Second, Baidu, Inc. may not be maximizing its static profits at the time and dynamic considerations are driving the observed low prices of Baidu and Baidu Map.

Column (3) presents the results of WeChat and Kwai. As we see in section 7.2, WeChat and Kwai are competing not only in functions but also in the time budget. After the merger, the comprehensive prices of WeChat and Kwai increase by 0.158 (1.7%) and 2.4 (61.7%). As a result, profits increase by about 146 yuan (0.4%) per week whereas consumers surplus decreases by 947 yuan (3.5%). By contrast, for 1000 users, consumer welfare decreases by 41 yuan per week after a merger of Baidu Map and Amap, a pair of direct competitors. The harm to consumers is large for a pair of seemingly independent apps.

The results suggest that without considering any dynamics, Tencent has incentive to acquire Kwai in the first quarter of 2017. After failed attempts to promote its own short-video app WeSee, Tencent invested $2 billion in Kwai in December 2019[47], and is now the largest institutional investor of Kwai after its IPO. Tencent also added short-video function to WeChat to directly compete with Kwai and Tik Tok in the first quarter of 2020.

8.4 Discussions

There are two different estimates of the ad revenue of Baidu Map in columns (1) and (2) in Table 16. The two different estimates are both "correct" because they are calculated with different assumptions. The ad revenue of Baidu Map in column (1) is backed out from the profit maximization condition of Baidu Map when Baidu is part of the generic app. The ad revenue of Baidu Map in column (2) is backed out from the profit maximization condition of Baidu Map when Amap is part of the generic app. Baidu and Amap can both shift the demand of Baidu Map. Ideally, I should include all relevant factors (substitutes, complements, and any other products in the same budget constraint) in a demand model. However, this is not realistic due to data availability and computational feasibility. This critique applies to all demand estimation.[48]

One can also consider subscription prices similar to (2). In a previous version of this paper, I simulate counterfactuals with both pay-per-use and subscription pricing.[49] Except for more simulation errors because of the discrete nature of subscription, the simulation results are similar. A notable difference is that profits and total surplus are always higher with pay-per-use. This is because users with higher willingness-to-pay (WTP) will pay more if they are charged per use. Pay-per-use enables firms to discriminate users based on usage.

[47] See https://www.scmp.com/tech/apps-social/article/3041747/tencent-said-invest-us2-billion-short-video-app-kuaishou

[48] See I.C in Gentzkow (2007) for a related discussion.

[49] This draft is available upon request.

Table 16: Mergers of the Three Pairs of Apps

	Baidu Map	Amap	Baidu	Baidu Map	WeChat	Kwai
	(1)		(2)		(3)	
Ad Revenue	3.45	3.78	5.12	3.63	9.57	3.89
Pre-Merger						
Prices	0	0	0.021	0.401	0	0
Active User	106	83	259	98	773	121
Total Usage	22.476	41.8	320.86	20.389	3739.57	202.201
Consumer Surplus	240.725		1453.065		27190.742	
Profits	235.546		1731.771		36574.234	
Total Surplus	476.271		3184.836		63764.976	
Post-Merger						
Prices	1.16	0.465	0	0	0.158	2.4
Active User	81	82	260	106	770	70
Total Usage	16.709	38.62	322.38	22.506	3706.16	106.16
Consumer Surplus	199.467		1468.462		26243.86	
Profits	240.953		1732.283		36720.006	
Total Surplus	440.421		3200.745		62963.866	

Note:
1, I simulate 1000 users with parameters estimated for an anonymous market in the first week of 2017.
2, All monetary values are in yuan.
3, The Total Usage variable is in hour.
Data Source: The author's calculations.

We do not often see firms employing pay-per-use because processing many small payments *ex post* is costly.

9 Conclusion

The rapid development of the mobile Internet industry and its profound influence on our society warrant further understanding of this industry. This paper informs the public debate on antitrust issues in the mobile Internet industry. In this paper, I develop a discrete-continuous model of consumer demand for apps that allows for complements as well as substitutes, and incorporates a binding time constraint. I estimate the model with a weekly panel of app usage in the first quarter of 2017 in China. I validate the model by applying it to three representative pairs of apps: each featured an important aspect of the competition landscape in this industry (*a priori* substitutes, *a priori* complements, and a pair with an ambiguous relationship). I then define and quantify budget competition. Budget competition can dominate functional competition and a merger of complementary apps can hurt consumers. I propose a simple model-free metric to gauge budget competition. When combined with a supply side, the estimated model can be used to simulate merger of apps.

The demand model in this paper incorporates four desirable features: discrete-continuous decisions, interactions between products, budget constraints, and estimation with instruments. This model can further incorporate other notable features in the mobile Internet industry (for example, advertisement and two-sidedness) or be adapted to study consumer demand for other goods and services. One shortcoming of the demand model is that it does not accommodate dynamics. Future work could consider modeling dynamics of apps.

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A Reduced Form Evidence of Identification

The identification of complemenatrity (γ_{12}) and correlated preference (ρ) is from two sources: the common user data and updates of apps as IV. In this section, I provide reduced-form evidence of their usefulness. Consider the following simple regression equation:

$$ln(q_{jt}) = a + b^{ols}_{jj'} ln(q_{j't}) + \varepsilon_1 \tag{28}$$

where q_{jt} is the number of active user of app j in week t in the whole nation. $b^{ols}_{jj'}$ summarizes the co-movement between j and j' and is increasing in both $\gamma_{jj'}$ and $\rho_{jj'}$. When we have common user data and updates, we can use the following two regressions:

$$ln(q_{jt} - c_{jj't}) = a + b^{c}_{jj'} ln(q_{j't} - c_{jj't}) + \varepsilon_2 \tag{29}$$

$$ln(q_{jt}) = a + b^{iv}_{jj'} ln(q_{j't}) + \varepsilon_1 \quad (30)$$

where $c_{jj't}$ is the number of common user between j and j'. In equation (30), I use the update history of the iOS version of j' as instruments for $q_{j't}$. Specifically, I use the cumulative numbers of small, medium, and major updates of j'. Therefore, $b^{diff}_{jj'} = b^{ols}_{jj'} - b^{c}_{jj'}$ is the information we can get from the common user data and $b^{bias}_{jj'} = b^{ols}_{jj'} - b^{iv}_{jj'}$ is the information we can get from the instruments. I then regress estimates of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'}, b^{diff}_{jj'}, b^{bias}_{jj'})$ on a category dummy which equals one if j and j' are in the same category defined by iResearch and zero otherwise. The categorization is based on functions and conforms to traditional definitions of a market (map, browser, music, etc.). Despite the criticism of categorizations in the introduction, they are still informative. A pair of apps in the same category should have a negative γ and a large ρ. If the estimates of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'}, b^{diff}_{jj'}, b^{bias}_{jj'})$ is correlated with this category dummy meaningfully, then we may conclude that the common user data and the IV are useful.

I have updates history of 84 apps and I run regressions for all $83 \times 84 = 6972$ pairs of apps. Note that I have only 13 observations for each pair of apps because both common user and update history are observed at the national level. I simulate 1000 samples of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'})$ for all pairs using the mean and variances from estimated equations (28), (29), and (30). Then I regress 1000 such samples of $(b^{ols}_{jj'}, b^{c}_{jj'}, b^{iv}_{jj'}, b^{diff}_{jj'}, b^{bias}_{jj'})$ on the category dummy. The mean and the confidence interval of the coefficients from the 1000 regressions are in Table 17.

The coefficient in column (1) of Table 17 is significant and positive. This is because preferences for apps in the same category are often highly correlated. Instruments should remove at least some of the bias because of correlated preferences (ρ). This is what we see in column (2): a smaller and insignificant coefficient. A cautionary note is that the coefficient in column (4) is not significant. The relationship between $b^{c}_{jj'}$ and the structural parameters $\gamma_{jj'}$ and $\rho_{jj'}$ is complicated. The co-movement of *the exclusive users* for apps in the same category is much larger than the co-movement of their *total active users*. One explanation is that the growth of competing apps mostly comes from exclusive users rather than common users. In other words, users who did not use A nor B started using A or B but not both. Overall, the reduced form results indicate that the common user data and updates are useful for our identification.

B Covariates in μ_1 and μ_2

The covaraites in μ_1 and μ_2 are the market fixed effects, week fixed effects. In Table 18, I provide covariates from the main specification of Baidu Map and Amap (Column (3) in Table 4). In the following table, I report week fixed effects and aggregate market fixed effects to gender and age groups. The results are reasonable: users between 31 and 35 and

Table 17: Reduced Form Evidence of Identification

	$b^{ols}_{jj'}$ (1)	$b^{iv}_{jj'}$ (2)	$b^{c}_{jj'}$ (3)	$b^{bias}_{jj'}$ (4)	$b^{diff}_{jj'}$ (5)
Same-Category	0.1636	0.0246	0.3784	0.139	-0.2148
	[0.087, 0.239]	[-0.269, 0.337]	[0.298, 0.456]	[-0.183, 0.45]	[-0.325, -0.103]
N	6972	6972	6972	6972	6972
R^2	0.0004	0.0000	0.0012	0.0001	0.0005

Note: The coefficients and the confidence interval are based on 1000 simulations.
Data Source: iResearch and the author's calculations.

male users derive higher utility from map apps because they are more likely to own and drive a car in China.

C Budget Competition

The intermediate bundle (t^i_0, t^i_1, t^i_3) defined by (8) is easy to calculate. The functional competition in Table **??** is $t^i_1 - t^o_1$. After this step, we can calculate how much time is left to be allocated as $\Delta T = T - t^i_0 - t^i_1 - t^i_3$. The intermediate bundle can be seen as the result of utility maximization over t_0, t_1, and t_3 subject to a time budget of $t^i_0 + t^i_1 + t^i_3$. Note that apps 0, 1, and 3 are independent. We can solve for the final bundle as the same utility maximization problem subject to a time budget of $t^i_0 + t^i_1 + t^i_3 + \Delta T$. The following two lemmas are useful when calculating the final bundle. The budget competition effect of app 2 on app 1 is $t^f_1 - t^i_1$.

Lemma 1. *For J independent apps that are used, when there are extra time ΔT, the increase in time spent on app j is $\Delta t_j = \Delta T \frac{1}{\gamma_j} \frac{1}{\sum_{k=1}^J \frac{1}{\gamma_k}}$.*

Proof. From the FOCs of the old bundle, we have

$$\mu_j + \gamma_j t^0_j = \mu_k + \gamma_k t^0_k \Rightarrow t^0_k = \frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t^0_j.$$

Similarily, we have $t^1_k = \frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t^1_j$. Because of the time constraint, we have

$$\sum_{l=1}^J t^0_l = T \Rightarrow \sum_{k=1}^J \left(\frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t^0_j\right) = T \Rightarrow t^0_j = \frac{T - \sum_k \frac{\mu_j - \mu_k}{\gamma_k}}{\gamma_j (\sum_{k=1}^J \frac{1}{\gamma_k})}$$

The budget constraint with extra time ΔT is

$$\sum_{l=1}^J t^1_l = T + \Delta T \Rightarrow \sum_{k=1}^J \left(\frac{\mu_j - \mu_k}{\gamma_k} + \frac{\gamma_j}{\gamma_k} t^1_j\right) = T + \Delta T \Rightarrow t^1_j = \frac{T + \Delta T - \sum \frac{\mu_j - \mu_k}{\gamma_k}}{\gamma_j (\sum_{k=1}^J \frac{1}{\gamma_k})}$$

Table 18: Covariates of Baidu Map and Amap in Taste Parameters

Covariates	Baidu Map	Standard Error	Amap	Standard Error
Week (02)	-0.0078	0.0002	-0.0452	0.0002
Week (03)	0.0275	0.0002	0.0026	0.0001
Week (04)	-0.0349	0.0000	-0.0466	0.0001
Week (05)	0.0276	0.0001	-0.0466	0.0002
Week (06)	0.0649	0.0002	-0.0466	0.0002
Week (07)	0.0631	0.0001	0.0053	0.0001
Week (08)	0.0657	0.0001	-0.0016	0.0002
Week (09)	0.0465	0.0001	-0.0138	0.0001
Week (10)	-0.0838	0.0002	-0.0138	0.0002
Week (11)	0.0944	0.0003	-0.0138	0.0004
Week (12)	0.107	0.0002	-0.0066	0.0003
Week (13)	0.1295	0.0004	-0.0138	0.0003
Female	0.265	-	0.4661	-
Male	0.3792	-	0.543	-
Age (<=24)	0.2557	-	0.5019	-
Age (25~30)	0.4124	-	0.6479	-
Age (31~35)	0.552	-	0.7084	-
Age (36~40)	0.2724	-	0.4142	-
Age (>=40)	0.1185	-	0.2504	-

Notes:
1, This table provides parameters of covariates in μ_1 and μ_2 corresponding to the column (3) of Table 4.
2, The coefficients of gender and age groups are the simple average of market fixed effects with corresponding characteristics.
Data Source: The author's calculations.

Therefore we have

$$\Delta t_j = t_j^1 - t_j^0 = \Delta T \frac{1}{\gamma_j} \frac{1}{\sum_{k=1}^{J} \frac{1}{\gamma_k}}$$

□

Lemma 2. *When an app q is used because of the extra time ΔT, $t_q^1 = \frac{T+\Delta T - \sum_k \frac{\mu_q-\mu_k}{\gamma_k}}{\gamma_q(\sum_k \frac{1}{\gamma_k})} \leq \frac{\Delta T}{\gamma_q(\sum_k \frac{1}{\gamma_k})}$*

Proof. Because q was not used ($t_q^0 = 0$), we have

$$\mu_q \leq \mu_k + \gamma_k t_k^0 \Rightarrow \frac{\mu_q - \mu_k}{\gamma_k} \geq t_k^0 \Rightarrow T \leq \sum_k \frac{\mu_q - \mu_k}{\gamma_k}$$

The FOCs of the new bundle are

$$\mu_q + \gamma_q t_q^1 = \mu_k + \gamma_k t_k^1 \Rightarrow t_k^1 = \frac{\mu_q - \mu_k}{\gamma_k} + \frac{\gamma_q}{\gamma_k} t_q^1$$

Combined with the new time constraint, we have

$$T + \Delta T = \sum_k \frac{\mu_q - \mu_k}{\gamma_k} + \frac{\gamma_q}{\gamma_k} t_q^1 \Rightarrow t_q^1 = \frac{T + \Delta T - \sum_{k \neq q} \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q (\sum_k \frac{1}{\gamma_k})}$$

Because $T \leq \sum_k \frac{\mu_j - \mu_k}{\gamma_k}$, we have

$$\frac{T + \Delta T - \sum_k \frac{\mu_q - \mu_k}{\gamma_k}}{\gamma_q (\sum_k \frac{1}{\gamma_k})} \leq \frac{\Delta T}{\gamma_q (\sum_k \frac{1}{\gamma_k})}.$$

□

D Derivation of the Model-Free Metric

The FOCs of this user at the observed usage level are

$$1 - 0.001 t_0^* = 1 - 0.001(T - t_1^* - t_2^*) = 1 + \gamma_1 t_1^* = 1 + \gamma_2 t_2^* \tag{31}$$

We have

$$t_1^* = \frac{0.001(t_2^* - T)}{\gamma_1 - 0.001} = \frac{\gamma_2}{\gamma_1} t_2^*$$

When app 2 exits the market, the new FOCs are

$$1 - 0.001 t_0' = 1 - 0.001(T - t_1') = 1 + \gamma_1 t_1'$$

We have

$$t_1' = \frac{-0.001T}{\gamma_1 - 0.001}$$

$$\begin{aligned} t_1' = \frac{-0.001T}{\gamma_1 - 0.001} &= \frac{-0.001T + 0.001 t_2^*}{\gamma_1 - 0.001} - \frac{0.001 t_2^*}{\gamma_1 - 0.001} \\ &= t_1^* - \frac{0.001 t_2^*}{\gamma_1 - 0.001} \\ &= t_1^* + t_2^* (\frac{-0.001}{\gamma_1 - 0.001}) \\ &= t_1^* + t_2^* (\frac{t_1^*}{t_0^* + t_1^*}) \end{aligned}$$

where the last equality is from $-0.001 t_0^* = \gamma_1 t_1^* = \gamma_2 t_2^*$.